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EFFECT OF TILLAGE DEPTH DIVISION AND VIBRATION ON SUBSOILER PERFORMANCE

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Abstract

The subsoiler plow is one of the most common types of plows that need a large amount of power as it used for tillage at great depths. so it must be characterized by the magnitude when it is manufactured to resistance the stresses from the soil and it is necessary to use blade with a large width that suits the depth needed to reach it. The width of blade should be not less than 16 cm when plowing at a depth of 80 cm. So that the plow can disturbed the soil well because when the width of blade is less than the value of 16 cm and used for plowing to a depth of 80 cm the blade causes lateral compaction of the soil layers. Which causes an increase in the compaction problem of the soil. Accordingly, the use of subsoiler plow requires significant power that lead to high costs for their use and difficulty in handling. Therefore, this research conducted to tillage at great depths with the highest efficiency in soil fragmentation and the lowest power requirements by manufacturing a subsoiler plow characterized by dividing the tillage depth into more than one depth using a number of tandem shanks with vibration. In addition, a theoretical mathematical model built on the Matlab program to predict the power requirements for the subsoiler plow in the case of dividing the tillage depth and vibration of plow shanks. The performance of subsoiler plow evaluated by conducting a field experiment in Sadat City, Menoufia Governorate, on sandy loam soil with an area of about one hectare. Field experiment treatments were, four levels to division the plowing depth 80 cm (one depth (80 cm) - two depths (40 cm) - three depths (27 cm) - four depths (20 cm)), three levels of distance between the plow shanks (15 cm - 30 cm - 45 cm) and two vibrating conditions of plow shanks (non-vibrating - vibrating). The results showed that in general division of the tillage depth of (80 cm) to more than the depth led to decrease the power requirements to pull subsoiler and fuel consumption rate. Especially when the tillage depth divided into four depths (20 cm), where the decreasing percentage was about (36% - 28%), respectively, compared to the use of one shank at a depth of (80cm). Likewise, the vibration of the plow shanks resulted in a decreasing the power requirements to pull subsoiler and fuel consumption rate about of (28% - 20%), respectively, compared to the non-vibration (fixed shanks). The results showed also, division of the tillage depth, especially in the case of division into four depths, caused a good soil fragmentation. Which, appeared in a decreasing in the soil bulk density and an increasing in the average infiltration rate of water through the soil layers about (13% - 36%), respectively, compared to using one shank at a depth of 80 cm. A decreasing in the soil bulk density and an increasing in the average infiltration rate of water were (5% - 10%), respectively, with the vibration of shanks compared to the fixed shanks. The optimum distance between tandem shanks of the plow was (25 cm), at which the lowest power consumption achieved. The mathematical model achieved great accuracy in predicting the power requirements of the plow when dividing the tillage depth and vibration of plow shanks where the R^2 was 0.94.

Keywords: power requirements of subsoiler, Tandem subsoiler, tillage depth division, vibration tillage.

Introduction

Energy and food are the major needs of most of the developing countries and some developed nations. In modern crop production systems, soil preparation for seed planting is the most expensive. To obtain a suitable seedbed, tillage equipments of various sizes and shapes are employed. Tillage is a basic practice in crop production system and it is defined as the mechanical manipulation of the soil in the tillage layer in order to promote the desired soil physical condition suitable for plant growth and development. The objectives of agricultural tillage are to provide a suitable environment for seed germination, root growth, weed control, soil erosion control, removal of compaction and stubble incorporation. Although tillage is an important operation in different soils for planting different crops, it has been regarded as the most energy intensive operation in a crop production system (Soekarno and Salokhe, 2005). Tillage accounting for almost fifty percent of the total energy consumed in crop production systems (Chi and Kushwaha, 1991). It is not always fully understood that the subsoilers do not perform equally well at all working depth. There is for every subsoiler a critical depth if exceeded, will cause compaction at depth rather than the required loosening. The soil disturbance above the critical depth is called a crescent failure, while the one below it, is called compression failure (Stafford, 1979). In crescent failure, the soil movement is forward, upward and sideways. In the compression failure, the soil movement is forward and sideways only. The critical depth occurred when the upward force exerted by the subsoiler on the soil equal to the soil confine pressure (Mckeys, 1985). Godwin and Spoor, (1977) concluded that the practical disadvantage of tilling below the critical depth is that the draft force increases while the soil disturbance reduces and soil compaction occurs. Spoor and Godwin, (1978) reported that the ratio of the tool-width to its working depth was found to influence the location of critical depth. At the same time, the position of the critical depth influences the maximum useful working depth of a tine. The practical disadvantage of working below the critical depth is that the draft force increases while soil disturbance is often reduced and soil compaction occurs. In the same study, it was reported that for effective soil loosening, crescent failure should occur. Furthermore, for a shallow working depth (i.e. above the critical depth), soil failure pattern was similar for different tine shapes but differ at a greater depth below the critical depth. Davis *et al.* (1982) reported that the critical depth should be pushed deep inside the soil to increase the crescent failure and reducing the compression failure and this can be done by loosening the top soil layers a head of the subsoiler using shallow tines attached to the frame of the subsoiler. Kasisira, (2004) proved that operating the tine below the critical depth reduces soil pulverization, causes soil compaction while increasing the soil resistance to the tine. Godwin and O'Dogherty, (2007) mentioned that transition from crescent to lateral failure occurs at (operating depth/tool width > 5). Hemmat *et al*. (2009) showed that soil failure does not occur in the same manner at all the operating depths as it depends on factors such as soil structure and critical depth. Abbas *et al*., (2012) reported that the critical depth at which the soil failure mode changed from brittle to compressive was at a working depth/shank width ratio of around five.

Hartge, (1988) noted that the least possible compaction within a whole soil profile can be assumed to be the one that is in equilibrium with the weight of the soil overlying it. This means that compaction and thus bulk density must increase with increasing depth below the soil surface. It follows therefore that even in virgin soils a compaction state prevails in subsoil layers. It will therefore be preferable to measure soil characteristics as influenced by depth and apply the values when testing the proposed force model. Srivastava *et al*. (1996) showed that subsoilers are operated at a greater depth than the other conventional tillage implements, to break up the hard subsoil layers, which result from compaction by traffic of farm equipment and tillage operations at the same shallow depth each season. They therefore have heavy shanks that can be operated at depths ranging between 450 to 850mm or deeper. Rahman and Chen, (2001) reported that the working depth of tillage implement was more critical than the working speed. Dahab and Mutwalli, (2002) reported that the traction force for chisel plough was higher in a soil of higher bulk density. Schwab *et al*. (2002) reported that in conservation tillage systems, yields might not be sustainable due to ill effects of soil compaction. Therefore, even in such systems, a deep tillage has to be used to ameliorate compacted soil profiles, even though the subsoiling process may disturb some of the valuable surface residue, hence reducing the benefits of conservation tillage. Mouazen and Ramon, (2002) reported that the draught force of a tillage implement increases with increasing bulk density. This holds true because the soil strength usually increases with increasing bulk density. Abo-Elnor *et al*. (2004) mentioned that subsoiling breaks up compaction layers, promotes aeration, limits runoff, and increases water holding capacity which helps soil retain moisture. It also alleviates the problem of excessive soil strength by reducing impedance to root penetration and improving root growth. And subsoiling can significantly enhance soil conservation, water infiltration, and crop yield has been proved. Kasisira, (2004) discovered that deep tilling once every number of years at the same depth actually increased the problem of compaction as a result of operating below the critical depth; he proposed an arrangement in which the tools are arranged in a tandem configuration to increasing efficiency of plow in disturbed soil process with reduced draft force. Mamman and Qui, (2005) studied the draft performance of a chisel plow model using a soil bin. The draft increased with increases in tillage depth. Mouazen and Ramon, (2006) reported that besides land, farm power is the second most important input to agricultural production. The most important factors in the determination of energy requirement of a tillage tool are draught and the amount of soil disturbed. Draughts of tillage tools and implements are mainly influenced by the physical and mechanical properties of soil, operating depth and speed, and tillage tool geometry. Tong and Moayad, (2006) used a computer simulation to predict soil-cutting parameters and implement power requirements of a chisel plough at different soil bulk densities. It was reported that the draught increased with increase in soil bulk density. Sahu and Raheman, (2006) conducted laboratory experiments to measure the draught requirements of a reference tillage tool, three scale-model individual and two combination tillage implements at different depths, speeds and cone index penetration resistances values in soil bin filled with sandy clay loam soil. Results showed that the total draught requirements of combination tillage implements and the draught utilization ratio of the rear passive set were significantly affected by depth, speed of operation and soil condition. It was also reported that the draught of all the tillage implements increased with increase in soil compaction, depth and speed of operation. Karoonboonyanan *et al*. (2007) pointed that tillage depth and tool speed are among the most prominent parameters that influence the draught of tillage tools and implements. Marenya, (2009) notes that an energy efficient tillage tool is that which accomplishes a particular tillage operation with reduced draft power requirement to overcome soil resistance; draft requirement is thus a reflection of the amount of soil resistance to a tillage tool. Naderloo *et al*. (2009) noted that the tillage operation requires the most energy and power spent on farms. Therefore, draft and power requirements are important in order to determine the size of the tractor that could be used for a specific implement. The draft required for a given implement will also be affected by the soil conditions and the geometry of the tillage implement. Shmulevich (2010) mentioned that an energy efficient tillage tool is that which accomplishes a particular tillage operation with reduced draft requirement to overcome soil resistance; draft requirement is thus a reflection of the amount of soil resistance to a tillage tool. Croitoru, (2015) showed that the subsoiler used to break open the hard pan the top soil. Usually, there are three layers of soil, name, top soil, hardpan and subsoil. The top soil may be few centimeters thick in which most of the cultivation for growing crop performed. The subsoiler improve soil structure by creating deep cracks and fissures in hard pan which results in better aeration, improves downward movement of water and deep rooting of the crops. Kadam and Chhapkhane, (2017) reported that besides that, subsoilers as a primary tillage equipment work in very arduous conditions, so they bear heavy dynamic loads. Therefore, proper design of these machines is necessary in order to increase their working life and reduce the farming costs due to high energy consumption.

Tanya and Salokhe, (2000) found that, the amount of soil fragments in the failure zone increased with the increase of tool the oscillating frequency. Niyamapa and Salokhe, (2000) reported that, forces acting on the vibration tillage tool decreased with an increase in oscillating frequency and oscillating amplitude. The soil surface was cracked due to tool motion showing the characteristics of lifting up of soil clods during the oscillating operation, whereas it showed the characteristics of soil flow during non-oscillating operation. The soil was pulverized more due to oscillating than non-oscillating operation. The reduction in bulk density of soil mass in the oscillating operation was about *30%* more than that during the non-oscillating mode. Gupta and Rajput, (2003) mentioned that, the oscillating tillage tool produced smaller soil aggregates than a non-oscillating. Linde, (2007) reports that the use of a vibrating tillage tool is an effective method of reducing the draft force. He utilized the vibratory mechanism to test and model the effect of the vibration on the draft force of a subsoiler. Joseph *et al*. (2007) reported that, applying vibratory motion in the longitudinal direction of a scaled bulldozer blade, a moldboard plow, and a chisel plow resulted in draft force reduction. Shchukin and Nagajka, (2015) reported that there are mainly two types of subsoilers: non-vibrating (conventional subsoiler), and vibrating (oscillating subsoiler). Vibrating subsoilers can reduce the draft force, but requires high power.

Godwin and Spoor, (1977) proposed a soil failure model with which the location of the critical depth can be determined. They further showed that the total draft force required to move an implement operating deeper than its critical depth, is a summation of the draft force required to fail soil above the critical depth with that required to fail soil below the critical depth. The method used by the above models to determine the total force on a tillage tool is rather complicated. It was simplified though by McKyes and Ali, (1977). Their model modified the soil failure ahead of a tool into a center failure wedge and two circular side crescents, and a plane failure surface at the bottom of the failed soil wedge that made it easier to solve the limit equilibrium equations. In this model, they incorporated an integration method, which evaluated the total force required to fail the side crescents as developed by Godwin, (1975). To simplify the integration process, the failure boundary on the surface is assumed to be circular. Based on Stafford, (1984) proposed dynamic models for both two and three-dimensional soil failure cases by introducing acceleration effects into these models. Following Perumpral's earlier research, Swick and Perumpral, (1988) proposed a three-dimensional dynamic soil failure model. The proposed soil failure zone is similar to the McKyes and Ali, (1977) static model and the force equation is derived in the same way except an acceleration force is included to account for the travel speed effect. They also modified the equation for determining the maximum width of the side circular wedge. Since this model accounts for acceleration force effects, it adequately predicted the forces encountered by a narrow tine. However it was found to over predict the soil-failure rapture radius. This was attributed to replacing the actual curved soil-failure plain with a straight one.

Therefore, to improve the subsoiler performance greatly in additional to the suitable soil conditions a mechanical modifications should be carried out on it. The mechanical modification is regarded the best method to improve the subsoiler performance. In this work, a modification was carried out on a conventional subsoiler of single tine. Four tines subsoiler were designed. The four tines arranged in a tandem configuration and provided with vibration movement. Thus, the objectives of this study were to measure the power requirements to pull subsoiler under varying conditions of tillage depth division with different vibration conditions of plow shanks and to measure soil disturbance parameters that arose from the experiments. In addition to development, the mathematical model of Swick and Perumpral, (1988) and built it on Matlab program to predict the power requirements to pull subsoiler under conditions of tillage depth division and vibration of plow shanks.

Materials and Methods

Subsoiler plow manufactured locally to tillage the soil at a depth of 80cm as shown in Figure (10) sketched side and elevation views and Figure (11) photography view. It is a mounted type hitched on the tractor using the three points hitching system. Whereas, this plow had the ability to tillage with one shank to a depth of 80 cm (conventional) and can be divide 80 cm tillage depth into, two depths (40 cm) using two tandem shanks, three depths (27 cm) using three tandem shanks and four depths (20 cm) using four tandem shanks as shown in Figure (13). In addition, the plow shanks can be vibration which driven by P.T.O shaft. Field experiment was conducted to performance evaluation of this plow in El-Sadat City, Menoufia Governorate at latitude: 30° 19' 5" N, longitude: 30° 32' 33" E, with sandy loam soil (Coarse sand 7.6%, Fine sand 51.4%, silt 18.4% and clay 22.6%) in November 2019 after harvesting of sorghum crop. Treatments were arranged in a split-split plot design with three replications. The Main plots were four levels to division the plowing depth 80 cm (one depth (80 cm) - two depths (40 cm) - three depths (27 cm) - four depths (20 cm)). The sub-plots were three levels of distance between the plow shanks (15 cm - 30 cm - 45 cm). The sub-sub plots were two vibrating conditions of plow shanks (non-vibrating - vibrating). All tillage treatments carried out at fixed operating speed (3.6 km/h or 1m/s) and power was transmitted to the input shaft of subsoiler from the tractor PTO (Power Take Off - 540 r/min) shaft through a universal joint, and then to the vibrators through the gearbox with a speed reduction. The Swick-Perumpral, (1988) mathematical force model was built on Matlab program to predict the power requirements to pull subsoiler at one depth 80cm and at division of tillage depth 80cm to two, three and four depths when plow shanks fixed or vibrated. The soil bulk density changes with the change in the soil depth and consequently the soil mechanical properties change with it, which leads to a change in the soil resistance, faced the plow blades with the change of the tillage depth. The mathematical model has been developed so that each study tillage depth has its own values of bulk density and mechanical properties (C - Φ) as shown in Table (1), which increases the accuracy and sensitivity of this model to predict the power requirements to pull subsoiler.

Table 1 : Bulk density and mechanical properties of soil at different study soil depths.

Soil internal friction angle, Φ (degrees)	Soil cohesion coefficient, C (N/m^2)	Soil bulk density, (N/m^3)	Soil depth, (cm)
15.42	12110	14130	(0-20)
15.84	12850	14610	(0-27)
16.74	14260	15210	(0-40)
17.31	14600	15370	(0-54)
18.23	15170	15810	(0-60)
19.77	16370	16830	(0-80)

Mathematical model for prediction the power requirements to pull subsoiler

Each plow has a tillage depth that fits its design specifications, called the critical depth, which is five times the width of the blade of plow (Godwin and O'Dogherty, 2007). When plowing within the limits of this critical depth, the plow works at the highest efficiency in terms of increasing the rate of soil disturbed and the lowest power requirements. If the tillage depth exceeds the critical depth, the plow's ability to disturbed the soil decreases and increases the pressure forces of the plow blade on the lateral soil layers and increases its compaction, thus increasing the power requirements. Therefore, the idea of this research is increase the efficiency of the performance of the subsoiler plow by dividing the deep tillage depth into a number of shallow depths (under the critical depth) using a number of tandem shanks configuration. So that the front blade (the first) raises a depth of soil on its surface, so the soil surface exposes in front of the next blade (the second), which raises another depth of soil on it, so the soil surface exposes to the next blade (the third), and so on for the following blades. However, it must be the distance between the tandem shanks is appropriate so that if this distance is less than necessary, there will not be enough time to allow the front blade to raise the soil and interference occurs between the blades, which leads to ineffective division of tillage depth. If the distance between the plows shanks is greater than necessary, the time will be large between entering the front and rear blades. Which causes the front blade to raise the soil and then drop it (disturbed soil) back to the surface of the soil before entering the rear blade to the soil, which leads to adding loads to the rear blade and increases the soil resistance as shown in Figure (1). Vibration of the plow shanks also made to increase the efficiency of soil disturbed and reduce the draft force. Mechanism of vibration system for subsoiler plow as shown in Figure (12). Although the vibration of the shanks consumes additional power, the power requirements to pull subsoiler with vibration is less than the power consumed in the case of non-vibration.

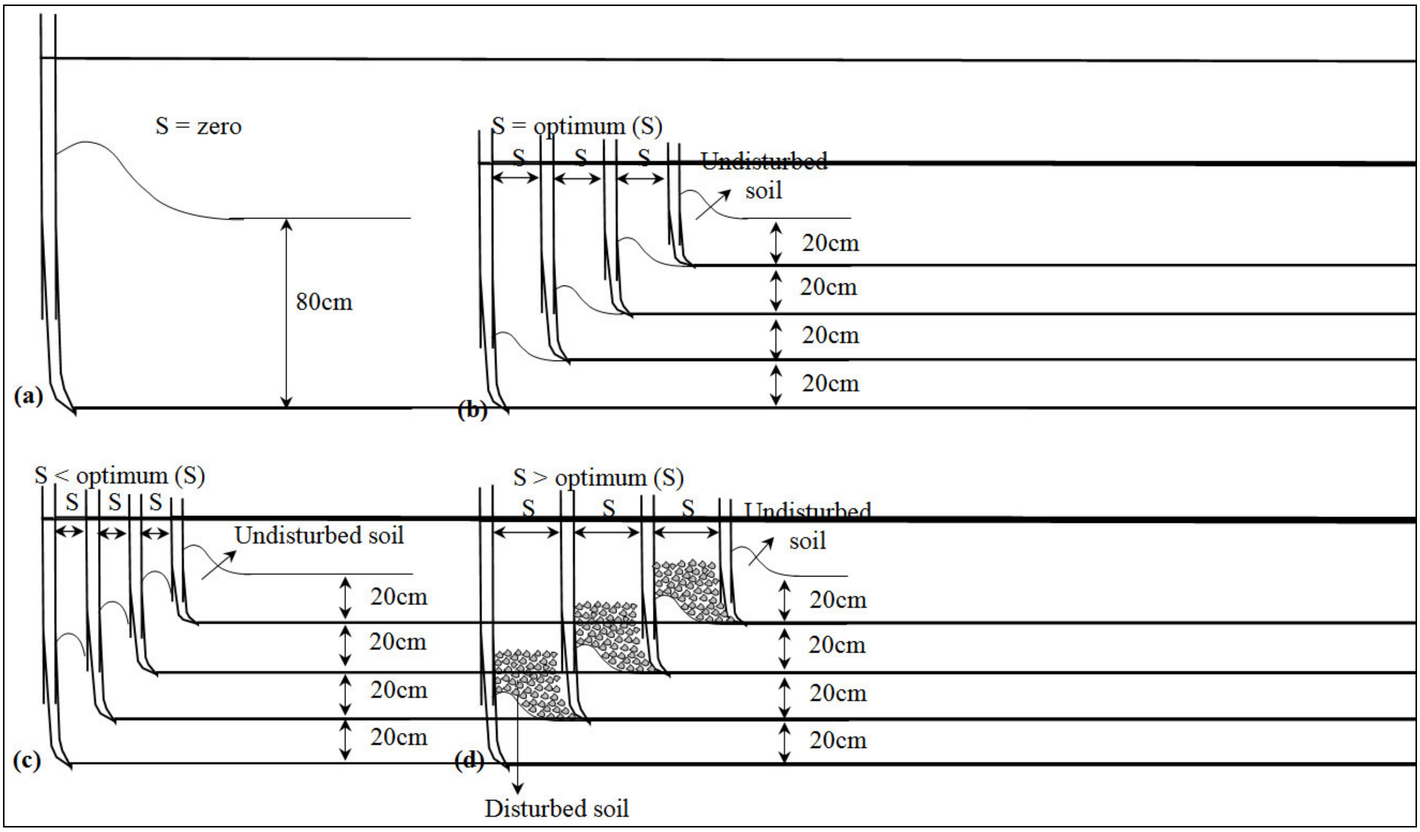

Fig. 1 : Effect of distance between plow shanks (S) on division of tillage depth operation when (a) S = zero, (b) S = optimum (S), (c) S < optimum (S) and (d) S > optimum (S).

A mathematical model (Swick-Perumpral, 1988) has been developed to calculate the power requirements to pull subsoiler to suit the conditions of dividing the tillage depth (two, three and four shanks in a tandem configuration) and the vibration of the plow shanks. Windows of inputs data for Matlab program to predict the power requirements to pull subsoiler presented in Figure (9). This mathematical model was built on the Matlab program and the components of the model were as following:

$ad_0 = C_0/2$
$ad_1 = C_1/2$
$ad_2 = C_2/2$
$ad_3 = C_3/2$
$ad_4 = C_4/2$
$\delta_0 = \tan^{-1}((\tan \Phi_0) / 2)$
$\delta_1 = \tan^{-1}((\tan \Phi_1) / 2)$
$\delta_2 = \tan^{-1}((\tan \Phi_2) / 2)$
$\delta_3 = \tan^{-1}((\tan \Phi_3) / 2)$
$\delta_4 = \tan^{-1}((\tan \Phi_4) / 2)$
$\beta_0 = 45 - (\Phi_0/2)$
$\beta_1 = 45 - (\Phi_1/2)$
$\beta_2 = 45 - (\Phi_2/2)$
$\beta_3 = 45 - (\Phi_3/2)$

$\beta_4 = 45 - (\Phi_4/2)$

$t = 2 * \pi / \blacksquare\ \omega$

$\omega_n = (k / m)^{0.5}$

$\blacksquare\ \Phi = \tan^{-1} ((((1 - \blacksquare\ \omega / \omega_n)^{0.5}) * (\blacksquare\ \omega / \omega_n)) / (1 - (\blacksquare\ \omega / \omega_n)^2))$

Total power requirements to pull subsoiler for the first shank, Ptt_1 (kW).

$R_{f1} = d_1 * (\tan \alpha + \tan \beta_1)$

$Fc_1 = \rho_1 * b * d_1 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_1)$

$Qc_1 = \rho_1 * b * d_1 * R_{f1}$

$Coc_1 = C_1 * b * d_1 / \sin \beta_1$

$Ca_1 = ad_1 * b * d_1 / \sin \alpha$

$Wc_1 = \rho_1 * b * d_1 * R_{f1} * 0.5$

$Hc_1 = (Qc_1 + Wc_1) * \sin (\Phi_1 + \beta_1) - Ca_1 * \cos (\alpha + \Phi_1 + \beta_1) + (Coc_1 + Fc_1) * \cos \Phi_1 / \sin (\alpha + \Phi_1 + \beta_1 + \delta_1)$

$P_{c1} = Hc_1 * v / 1000$

$S_{f1} = (46 * R_{f1} + \sin \alpha * 0.904 - 6.03) / 100$

$\theta_1 = \sin^{-1} (S_{f1} / R_{f1})$

$Fs_1 = \rho_1 * d_1 * R_{f1} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_1)$

$Qs_1 = \rho_1 * d_1 * R_{f1}^2 /2$

$Cos_1 = C_1 * d_1 * R_{f1} / 2 * \sin \beta_1$

$Ws_1 = \rho_1 * d_1 * R_{f1}^2 /6$

$Hs_1 = ((Qs_1 + Ws_1) * \sin (\Phi_1 + \beta_1) * \sin \theta_1 + Fs_1 * \cos \Phi_1 * \theta_1 / 2 + \sin 2\theta_1 / 4 + Cos_1 * \cos \Phi_1 * \sin \theta_1)) / \sin (\alpha + \Phi_1 + \beta_1 + \delta_1)$

$Ps_1 = Hs_1 * v / 1000$

$Ptt_1 = P_{c1} + 2 * Ps_1$

Net power requirements to pull subsoiler for the second shank when S = 15-30cm, Ptt_2 (kW).

$R_{f2} = d_2 * (\tan \alpha + \tan \beta_2)$

$Fc_2 = \rho_2 * b * d_2 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_2)$

$Qc_2 = \rho_2 * b * d_2 * R_{f2}$

$Coc_2 = C_2 * b * d_2 / \sin \beta_2$

$Ca_2 = ad_2 * b * d_2 / \sin \alpha$

$Wc_2 = \rho_2 * b * d_2 * R_{f2} * 0.5$

$Hc_2 = (Qc_2 + Wc_2) * \sin (\Phi_2 + \beta_2) - Ca_2 * \cos (\alpha + \Phi_2 + \beta_2) + (Coc_2 + Fc_2) * \cos \Phi_2 / \sin (\alpha + \Phi_2 + \beta_2 + \delta_2)$

$P_{c2} = Hc_2 * v / 1000$

$S_{f2} = (46 * R_{f2} + \sin \alpha * 0.904 - 6.03) / 100$

$\theta_2 = \sin^{-1} (S_{f2} / R_{f2})$

$Fs_2 = \rho_2 * d_2 * R_{f2} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_2)$

$Qs_2 = \rho_2 * d_2 * R_{f2}^2 /2$

$Cos_2 = C_2 * d_2 * R_{f2} / 2 * \sin \beta_2$

$Ws_2 = \rho_2 * d_2 * R_{f2}^2 /6$

$Hs_2 = ((Qs_2 + Ws_2) * \sin (\Phi_2 + \beta_2) * \sin \theta_2 + Fs_2 * \cos \Phi_2 * \theta_2 / 2 + \sin 2\theta_2 / 4 + Cos_2 * \cos \Phi_2 * \sin \theta_2)) / \sin (\alpha + \Phi_2 + \beta_2 + \delta_2)$

$Ps_2 = Hs_2 * v / 1000$

$Pt_2 = P_{c2} + 2 * Ps_2$

$R_{f22} = d_1 * (\tan \alpha + \tan \beta_1)$

$Fc_{22} = \rho_1 * b * d_1 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_1)$

$Qc_{22} = \rho_1 * b * d_1 * R_{f22}$

$Coc_{22} = C_1 * b * d_1 / \sin \beta_1$

$Ca_{22} = ad_1 * b * d_1 / \sin \alpha$

$Wc_{22} = \rho_1 * b * d_1 * S$

$Hc_{22} = (Qc_{22} + Wc_{22}) * \sin (\Phi_1 + \beta_1) - Ca_{22} * \cos (\alpha + \Phi_1 + \beta_1) + (Coc_{22} + Fc_{22}) * \cos \Phi_1 / \sin (\alpha + \Phi_1 + \beta_1 + \delta_1)$

$P_{c22} = Hc_{22} * v / 1000$

$S_{f22} = (46 * R_{f22} + \sin \alpha * 0.904 - 6.03) / 100$

$\theta_{22} = \sin^{-1} (S_{f22} / R_{f22})$

$Fs_{22} = \rho_1 * d_1 * R_{f22} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_1)$

$Qs_{22} = \rho_1 * d_1 * R_{f22}^2 /2$

$Cos_{22} = C_1 * d_1 * R_{f22} / 2 * \sin \beta_1$

$Ws_{22} = 0.5 * \rho_1 * d_1 * R_{f22} * S * \sin \theta_{22}$

$Hs_{22} = ((Qs_{22} + Ws_{22}) * \sin (\Phi_1 + \beta_1) * \sin \theta_{22} + Fs_{22} * \cos \Phi_1 * \theta_{22} / 2 + \sin 2\theta_{22} / 4 + Cos_{22} * \cos \Phi_1 * \sin \theta_{22})) / \sin (\alpha + \Phi_1 + \beta_1 + \delta_1)$

$Ps_{22} = Hs_{22} * v / 1000$

$Pt_{22} = P_{c22} + 2 * Ps_{22}$

$Ptt_2 = Pt_2 - (Ptt_1 + Pt_{22})$

Net power requirements to pull subsoiler for the third shank when S = 15-30cm, Ptt_3 (kW).

$R_{f3} = d_3 * (\tan \alpha + \tan \beta_3)$

$Fc_3 = \rho_3 * b * d_3 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_3)$

$Qc_3 = \rho_3 * b * d_3 * R_{f3}$

$Coc_3 = C_3 * b * d_3 / \sin \beta_3$

$Ca_3 = ad_3 * b * d_3 / \sin \alpha$

$Wc_3 = \rho_3 * b * d_3 * R_{f3} * 0.5$

$Hc_3 = (Qc_3 + Wc_3) * \sin (\Phi_3 + \beta_3) - Ca_3 * \cos (\alpha + \Phi_3 + \beta_3) + (Coc_3 + Fc_3) * \cos \Phi_3 / \sin (\alpha + \Phi_3 + \beta_3 + \delta_3)$

$P_{c3} = Hc_3 * v / 1000$

$S_{f3} = (46 * R_{f3} + \sin \alpha * 0.904 - 6.03) / 100$

$\theta_3 = \sin^{-1} (S_{f3} / R_{f3})$

$Fs_3 = \rho_3 * d_3 * R_{f3} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_3)$

$Qs_3 = \rho_3 * d_3 * R_{f3}^2 /2$

$Cos_3 = C_3 * d_3 * R_{f3} / 2 * \sin \beta_3$

$Ws_3 = \rho_3 * d_3 * R_{f3}^2 /6$

$Hs_3 = ((Qs_3 + Ws_3) * \sin (\Phi_3 + \beta_3) * \sin \theta_3 + Fs_3 * \cos \Phi_3 * \theta_3 / 2 + \sin 2\theta_3 / 4 + Cos_3 * \cos \Phi_3 * \sin \theta_3)) / \sin (\alpha + \Phi_3 + \beta_3 + \delta_3)$

$Ps_3 = Hs_3 * v / 1000$

$Pt_3 = P_{c3} + 2 * Ps_3$

$R_{f33} = d_2 * (\tan \alpha + \tan \beta_2)$

$Fc_{33} = \rho_2 * b * d_2 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_2)$

$Qc_{33} = \rho_2 * b * d_2 * R_{f33}$

$Coc_{33} = C_2 * b * d_2 / \sin \beta_2$

$Ca_{33}= ad_2 * b * d_2 / \sin \alpha$

$Wc_{33}= \rho_2 * b * d_2 * S$

$Hc_{33}= (Qc_{33} + Wc_{33}) * \sin (\Phi_2 + \beta_2) – Ca_{33} * \cos (\alpha + \Phi_2 + \beta_2) + (Coc_{33} + Fc_{33}) * \cos \Phi_2 / \sin (\alpha + \Phi_2 + \beta_2 + \delta_2)$

$P_{c33}= Hc_{33} * v / 1000$

$S_{f33}= (46 * R_{f33} + \sin \alpha * 0.904 – 6.03) / 100$

$\theta_{33}= \sin^{-1} (S_{f33} / R_{f33})$

$Fs_{33}= \rho_2 * d_2 * R_{f33} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_2)$

$Qs_{33}= \rho_2 * d_2 * R_{f33}^2 /2$

$Cos_{33}= C_2 * d_2 * R_{f33} / 2 * \sin \beta_2$

$Ws_{33}= 0.5 * \rho_2 * d_2 * R_{f33} *S * \sin \theta_{33}$

$Hs_{33} = ((Qs_{33} + Ws_{33}) * \sin (\Phi_2+ \beta_2) * \sin \theta_{33} + Fs_{33} * \cos \Phi_2 * \theta_{33} / 2 + \sin 2 \theta_{33} / 4 + Cos_{33} * \cos \Phi_2 * \sin \theta_{33})) / \sin (\alpha + \Phi_2 + \beta_2 + \delta_2)$

$Ps_{33}= Hs_{33} * v / 1000$

$Pt_{33}= P_{c33} + 2 * Ps_{33}$

$Ptt_3= Pt_3 – (Pt_2 + Pt_{33})$

Net power requirements to pull subsoiler for the fourth shank when S = 15-30cm, Ptt_4 (kW).

$R_{f4}= d_4 * (\tan \alpha + \tan \beta_4)$

$Fc_4= \rho_4 * b * d_4 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_4)$

$Qc_4= \rho_4 * b * d_4* R_{f4}$

$Coc_4= C_4 * b * d_4 / \sin \beta_4$

$Ca_4= ad_4 * b * d_4 / \sin \alpha$

$Wc_4= \rho_4 * b * d_4 * R_{f4} * 0.5$

$Hc_4= (Qc_4 + Wc_4) * \sin (\Phi_4 + \beta_4) – Ca_4 * \cos (\alpha + \Phi_4 + \beta_4) + (Coc_4 + Fc_4) * \cos \Phi_4 / \sin (\alpha + \Phi_4 + \beta_4 + \delta_4)$

$P_{c4}= Hc_4 * v / 1000$

$S_{f4}= (46 * R_{f4} + \sin \alpha * 0.904 – 6.03) / 100$

$\theta_4= \sin^{-1} (S_{f4} / R_{f4})$

$Fs_4= \rho_4 * d_4 * R_{f4} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_4)$

$Qs_4= \rho_4 * d_4 * R_{f4}^2 /2$

$Cos_4= C_4 * d_4 * R_{f4} / 2 * \sin \beta_4$

$Ws_4= \rho_4 * d_4 * R_{f4}^2 /6$

$Hs_4= ((Qs_4 + Ws_4) * \sin (\Phi_4 + \beta_4) * \sin \theta_4 + Fs_4 * \cos \Phi_4 * \theta_4 / 2 + \sin 2 \theta_4 / 4 + Cos_4 * \cos \Phi_4 * \sin \theta_4)) / \sin (\alpha + \Phi_4 + \beta_4 + \delta_4)$

$Ps_4= Hs_4 * v / 1000$

$Pt_4= P_{c4} + 2 * Ps_4$

$R_{f44}= d_3 * (\tan \alpha + \tan \beta_3)$

$Fc_{44}= \rho_3 * b * d_3 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_3)$

$Qc_{44}= \rho_3 * b * d_3 * R_{f44}$

$Coc_{44}= C_3 * b * d_3 / \sin \beta_3$

$Ca_{44}= ad_3 * b * d_3 / \sin \alpha$

$Wc_{44}= \rho_3 * b * d_3 * S$

$Hc_{44}= (Qc_{44} + Wc_{44}) * \sin (\Phi_3 + \beta_3) – Ca_{44} * \cos (\alpha + \Phi_3 + \beta_3) + (Coc_{44} + Fc_{44}) * \cos \Phi_3 / \sin (\alpha + \Phi_3 + \beta_3 + \delta_3)$

$P_{c44}= Hc_{44} * v / 1000$

$S_{f44}= (46 * R_{f44} + \sin \alpha * 0.904 – 6.03) / 100$

$\theta_{44}= \sin^{-1} (S_{f44} / R_{f44})$

$Fs_{44}= \rho_3 * d_3 * R_{f44} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_3)$

$Qs_{44}= \rho_3 * d_3 * R_{f44}^2 /2$

$Cos_{44}= C_3 * d_3 * R_{f44} / 2 * \sin \beta_3$

$Ws_{44}= 0.5 * \rho_3 * d_3 * R_{f44} *S * \sin \theta_{44}$

$Hs_{44}= ((Qs_{44} + Ws_{44}) * \sin (\Phi_3+ \beta_3) * \sin \theta_{44} + Fs_{44} * \cos \Phi_3 * \theta_{44} / 2 + \sin 2 \theta_{44} / 4 + Cos_{44} * \cos \Phi_3 * \sin \theta_{44})) / \sin (\alpha + \Phi_3 + \beta_3 + \delta_3)$

$Ps_{44}= Hs_{44} * v / 1000$

$Pt_{44}= P_{c44} + 2 * Ps_{44}$

$Ptt_4= Pt_4 – (Pt_3 + Pt_{44})$

Total power requirements to fail the disturbed wedge between the first and second shank when, S= 45cm, Ptt_5 (kW).

$R_{f01}= d_1 * (\tan \alpha + \tan \beta_0)$

$Fc_{01}= \rho_0 * b * d_1 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_0)$

$Qc_{01}= \rho_0 * b * d_1 * R_{f01}$

$Coc_{01}= C_0 * b * d_1 / \sin \beta_0$

$Ca_{01}= ad_0 * b * d_1 / \sin \alpha$

$Wc_{01}= \rho_0 * b * d_1 * S$

$Hc_{01}= (Qc_{01} + Wc_{01}) * \sin (\Phi_0 + \beta_0) - Ca_{01} * \cos (\alpha + \Phi_0 + \beta_0) + (Coc_{01} + Fc_{01}) * \cos \Phi_0 / \sin (\alpha + \Phi_0 + \beta_0 + \delta_0)$

$P_{c01}= Hc_{01} * v / 1000$

$S_{f01}= (46 * R_{f01} + \sin \alpha * 0.904 – 6.03) / 100$

$\theta_{01}= \sin^{-1} (S_{f01} / R_{f01})$

$Fs_{01}= \rho_0 * d_1 * R_{f01} * v^2 * \sin \alpha / 2 * g * \sin (\alpha + \beta_0)$

$Qs_{01}= \rho_0 * d_1 * R_{f01}^2 /2$

$Cos_{01}= C_0 * d_1 * R_{f01} / 2 * \sin \beta_0$

$Ws_{01}= 0.5 * \rho_0 * d_1 * R_{f01} *S * \sin \theta_{01}$

$Hs_{01} = ((Qs_{01} + Ws_{01}) * \sin (\Phi_0+ \beta_0) * \sin \theta_{01} + Fs_{01} * \cos \Phi_0 * \theta_{01} / 2 + \sin 2 \theta_{01} / 4 + Cos_{01} * \cos \Phi_0 * \sin \theta_{01})) / \sin (\alpha + \Phi_0 + \beta_0 + \delta_0)$

$Ps_{01}= Hs_{01} * v / 1000$

$Ptt_5= P_{c01} + 2 * Ps_{01}$

Total power requirements to fail the disturbed wedge between the second and third shank when, S= 45cm, Ptt_6 (kW).

$R_{f02}= d_2 * (\tan \alpha + \tan \beta_0)$

$Fc_{02}= \rho_0 * b * d_2 * v^2 * \sin \alpha / g * \sin (\alpha + \beta_0)$

$Qc_{02}= \rho_0 * b * d_2 * R_{f02}$

$Coc_{02}= C_0 * b * d_2 / \sin \beta_0$

$Ca_{02}= ad_0 * b * d_2 / \sin \alpha$

$Wc_{02}= \rho_0 * b * d_2 * S$

$Hc_{02}= (Qc_{02} + Wc_{02}) * \sin (\Phi_0 + \beta_0) - Ca_{02} * \cos (\alpha + \Phi_0 + \beta_0) + (Coc_{02} + Fc_{02}) * \cos \Phi_0 / \sin (\alpha + \Phi_0 + \beta_0 + \delta_0)$

$P_{c02}= Hc_{02} * v / 1000$

$S_{f02} = (46 * R_{f02} + \sin\alpha * 0.904 - 6.03) / 100$

$\theta_{02} = \sin^{-1}(S_{f02} / R_{f02})$

$Fs_{02} = \rho_0 * d_2 * R_{f02} * v^2 * \sin\alpha / 2 * g * \sin(\alpha + \beta_0)$

$Qs_{02} = \rho_0 * d_2 * R_{f02}^2 / 2$

$Cos_{02} = C_0 * d_2 * R_{f02} / 2 * \sin\beta_0$

$Ws_{02} = 0.5 * \rho_0 * d_2 * R_{f02} * S * \sin\theta_{02}$

$Hs_{02} = ((Qs_{02} + Ws_{02}) * \sin(\Phi_0 + \beta_0) * \sin\theta_{02} + Fs_{02} * \cos\Phi_0 * \theta_{02} / 2 + \sin 2\theta_{02} / 4 + Cos_{02} * \cos\Phi_0 * \sin\theta_{02})) / \sin(\alpha + \Phi_0 + \beta_0 + \delta_0)$

$Ps_{02} = Hs_{02} * v / 1000$

$Ptt_6 = P_{c02} + 2 * Ps_{02}$

Total power requirements to fail the disturbed wedge between the third and fourth shank when, S= 45cm, Ptt_7 (kW).

$R_{f03} = d_3 * (\tan\alpha + \tan\beta_0)$

$Fc_{03} = \rho_0 * b * d_3 * v^2 * \sin\alpha / g * \sin(\alpha + \beta_0)$

$Qc_{03} = \rho_0 * b * d_3 * R_{f03}$

$Coc_{03} = C_0 * b * d_3 / \sin\beta_0$

$Ca_{03} = ad_0 * b * d_3 / \sin\alpha$

$Wc_{03} = \rho_0 * b * d_3 * S$

$Hc_{03} = (Qc_{03} + Wc_{03}) * \sin(\Phi_0 + \beta_0) - Ca_{03} * \cos(\alpha + \Phi_0 + \beta_0) + (Coc_{03} + Fc_{03}) * \cos\Phi_0 / \sin(\alpha + \Phi_0 + \beta_0 + \delta_0)$

$P_{c03} = Hc_{03} * v / 1000$

$S_{f03} = (46 * R_{f03} + \sin\alpha * 0.904 - 6.03) / 100$

$\theta_{03} = \sin^{-1}(S_{f03} / R_{f03})$

$Fs_{03} = \rho_0 * d_3 * R_{f03} * v^2 * \sin\alpha / 2 * g * \sin(\alpha + \beta_0)$

$Qs_{03} = \rho_0 * d_3 * R_{f03}^2 / 2$

$Cos_{03} = C_0 * d_3 * R_{f03} / 2 * \sin\beta_0$

$Ws_{03} = 0.5 * \rho_0 * d_3 * R_{f03} * S * \sin\theta_{03}$

$Hs_{03} = ((Qs_{03} + Ws_{03}) * \sin(\Phi_0 + \beta_0) * \sin\theta_{03} + Fs_{03} * \cos\Phi_0 * \theta_{03} / 2 + \sin 2\theta_{03} / 4 + Cos_{03} * \cos\Phi_0 * \sin\theta_{03})) / \sin(\alpha + \Phi_0 + \beta_0 + \delta_0)$

$Ps_{03} = Hs_{03} * v / 1000$

$Ptt_7 = P_{c03} + 2 * Ps_{03}$

a- Power requirements to pull subsoiler for subsoiler (one shank)

1- Power requirements to pull subsoiler without vibration.

$PT_1 = Ptt_1 = P_{c1} + 2 * Ps_1$

2- Power requirements with vibration.

$PT_2 = PT_1 + PT_1 (1/v) * (v - 2 * 1/t * \sin \cdot \Phi)$

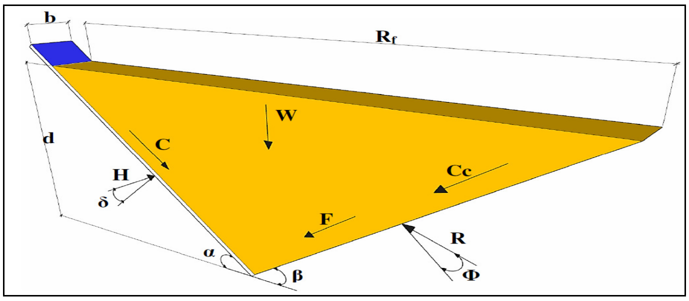

Fig. 2 : Soil center wedge failed by one shank.

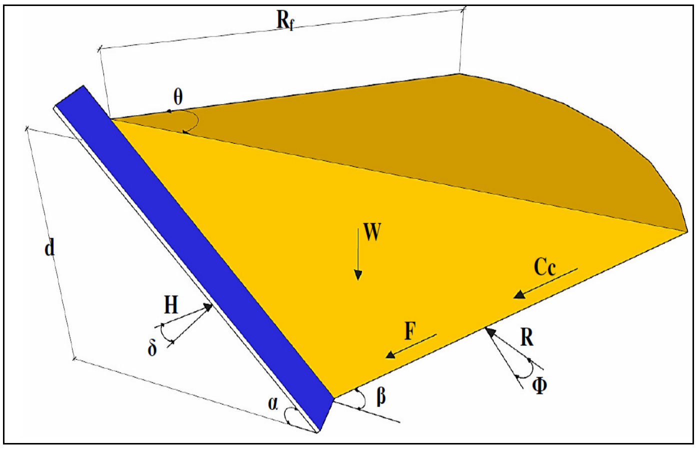

Fig. 3 : Half section of soil side circular wedge failed by one shank.

b- Power requirements to pull subsoiler (tandem shanks).

- **Power requirements to pull subsoiler (two shanks).**

1- Power requirements to pull subsoiler two shanks when, S= 15-30cm without vibration, PT_3 (kW).

$PT_3 = Ptt_1 + Ptt_2$

2- Power requirements to pull subsoiler two shanks when, S= 45cm without vibration, PT_4 (kW).

$PT_4 = Ptt_1 + Ptt_2 + Ptt_5$

3- Power requirements to pull subsoiler two shanks when, S= 15-30cm with vibration, PT_5 (kW).

$PT_5 = PT_3 + PT_3 (1/v) * (v - 2 * 1/t * \sin \cdot \Phi)$

4- Power requirements to pull subsoiler two shanks when, S= 45cm with vibration, PT_6 (kW).

$PT_6 = PT_4 + PT_4 (1/v) * (v - 2 * 1/t * \sin \cdot \Phi)$

- **Power requirements to pull subsoiler (three shanks).**

1- Power requirements to pull subsoiler two shanks when, S= 15-30cm without vibration, PT_7 (kW).

$PT_7 = Ptt_1 + Ptt_2 + Ptt_3$

2- Power requirements to pull subsoiler two shanks when, S= 45cm without vibration, PT_8 (kW).

$PT_8 = Ptt_1 + Ptt_2 + Ptt_3 + Ptt_5 + Ptt_6$

3- Power requirements to pull subsoiler two shanks when, S= 15-30cm with vibration, PT_9 (kW).

$PT_9 = PT_7 + PT_7 (1/v) * (v - 2 * 1/t * \sin \cdot \Phi)$

4- Power requirements to pull subsoiler two shanks when, S= 45cm with vibration, PT_{10} (kW).

$PT_{10} = PT_8 + PT_8 (1/v) * (v - 2 * 1/t * \sin \cdot \Phi) + (\pi * F_0 * 1/t * \sin \cdot \Phi)$

- **Power requirements to pull subsoiler (four shanks).**

1- Power requirements to pull subsoiler two shanks when, S= 15-30cm without vibration, PT_{11} (kW).

$PT_{11} = Ptt_1 + Ptt_2 + Ptt_3 + Ptt_4$

2- Power requirements to pull subsoiler two shanks when, S= 45cm without vibration, PT_{12} (kW).

$PT_{12} = Ptt_1 + Ptt_2 + Ptt_3 + Ptt_4 + Ptt_5 + Ptt_6 + Ptt_7$

3- Power requirements to pull subsoiler two shanks when, S= 15-30cm with vibration, PT_{13} (kW).

$$PT_{13} = PT_{11} + PT_{11}\ (1/v) * (v - 2 * 1/t * \sin \cdot \Phi)$$

4- Power requirements to pull subsoiler two shanks when, S= 45cm with vibration, PT_{14} (kW).

$$PT_{14} = PT_{12} + PT_{12}\ (1/v) * (v - 2 * 1/t * \sin \cdot \Phi).$$

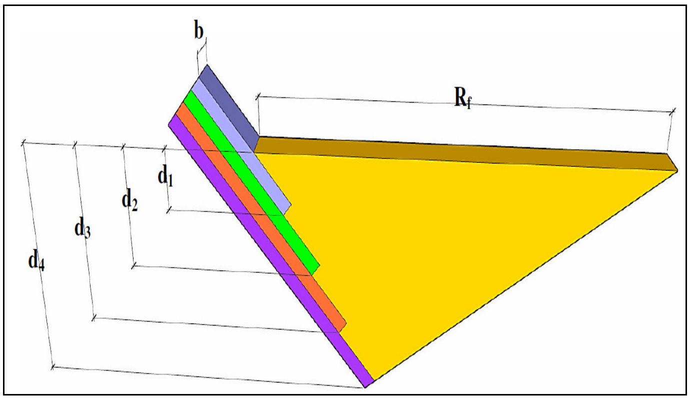

Fig. 4 : Soil failed by four shanks at distance between shanks, S = Zero m.

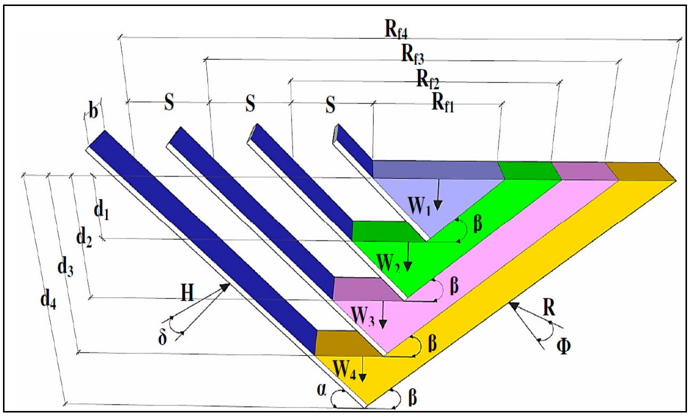

Fig. 5 : Soil center wedge failed by four shanks at distance between shanks, S = 0.15-0.3 m.

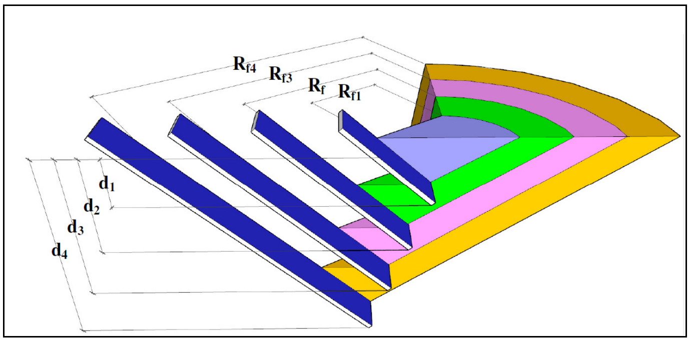

Fig. 6 : Half section of soil side circular wedge failed by four shanks at distance between shanks, S = 0.15-0.3 m

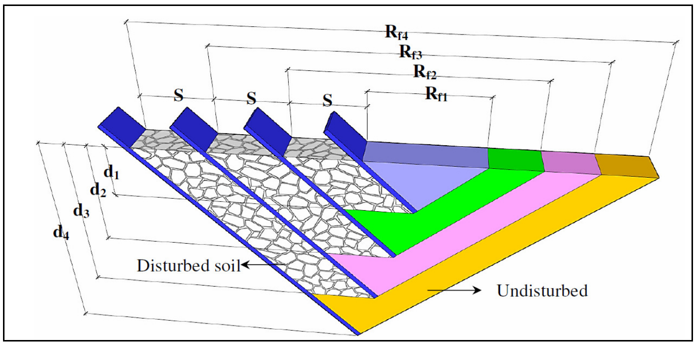

Fig. 7 : Soil center wedge failed by four shanks at distance between shanks, S = 0.45 m.

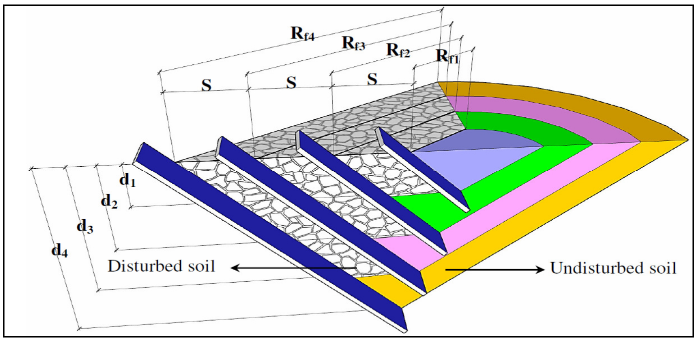

Fig. 8 : Half section of soil side circular wedge failed by four shanks at distance between shanks, S = 0.45 m

		Where:
m	Subsoiler operating depth,	d_1, d_2, d_3 ***and*** d_4
N.m^{-3}	Soil unit weight of disturbed soil,	ρ_0
N.m^{-3}	Soil unit weight of undisturbed soil at d_1, d_2, d_3 and d_4 soil depth respectively,	ρ_1, ρ_2, ρ_3 ***and*** ρ_4
degrees	Soil internal friction angle of disturbed soil,	Φ_0
degrees	Soil internal friction angle of undisturbed soil at d_1, d_2, d_3 and d_4 soil depth respectively,	Φ_1, Φ_2, Φ_3 ***and*** Φ_4
N.m^{-2}	Cohesion coefficient of disturbed soil,	C_0
N.m^{-2}	Cohesion coefficient of undisturbed soil at d_1, d_2, d_3 and d_4 soil depth respectively,	C_1, C_2, C_3 ***and*** C_4
N.m^{-2}	Adhesion factor of disturbed soil,	ad_0
N.m^{-2}	Adhesion factor of undisturbed soil at d_1, d_2, d_3 and d_4 soil depth respectively,	ad_1, ad_2, ad_3 ***and*** ad_4
N	Adhesion force at the interface,	***Ca***
m.s^{-2}	Gravitational constant, 9.81,	***g***
degrees	Rake angle,	α
m	Tool width,	***b***
degrees	Angle between the soil rupture plain and the horizontal soil surface of disturbed soil,	β_0
degrees	Angle between the soil rupture plain and the horizontal soil surface of undisturbed soil at d_1, d_2, d_3 and d_4 soil depth respectively,	$\beta_1, \beta_2, \beta_3$ ***and*** β_4
degrees	Interface friction angle of disturbed soil,	δ_0
degrees	Interface friction angle of undisturbed soil at d_1, d_2, d_3 and d_4 soil depth respectively,	$\delta_1, \delta_2, \delta_3$ ***and*** δ_4
N	Acceleration force the center wedge,	***Fc***
N	Acceleration force in the side circular wedge,	***Fs***
N	Surcharge force at the center wedge,	***Qc***
N	Surcharge force at the side circular wedge,	***Qs***
N	Soil weight failed in the center wedge,	***Wc***

N	Soil weight failed in the side circular wedge,	***Ws***
N	Cohesion force at the rupture plain at the center wedge,	***Coc***
N	Cohesion force at the rupture plain at the side wedge,	***Cos***
m	Soil rupture radius due to the subsoiler,	R_{f1},R_{f2}, R_{f3} and R_{f4}
N	Reaction force at the rupture plane,	***R***
$m.s^{-1}$	Operating speed,	***v***
m	Projected distance between the shanks,	***S***
degrees	Horizontal included angle of the circular side crescent,	θ
m	Oscillatory amplitude,	***L***
kg	Mass,	***m***
$N.m^{-1}$	Stiffness of the frame,	***k***
rad/s	Driving force frequency,	ω
rad/s	Undamped natural frequency of system,	ω_n
degrees	Phase angle,	Φ
s	Time,	***t***
N	Draft force to fail center portion of the failure wedge,	***Hc***
N	Draft force to fail the side circular wedge,	***Hs***
m	Maximum width of the circular side wedge,	S_f
kW	Power requirements to fail the center wedge for the first shank,	P_{c1}
kW	Power requirements to fail the side circular wedge for the first shank,	P_{s1}
kW	Total power requirements for the first shank,	Ptt_1
kW	Power requirements to fail the center wedge for the second shank,	P_{c2}
kW	Power requirements to fail the side circular wedge for the second shank,	P_{s2}
kW	Total power requirements for the second shank**,**	Pt_2
kW	Power requirements to fail the center wedge between the first and second shank,	P_{c22}
kW	Power requirements to fail the side circular wedge between the first and second shank,	P_{s22}
kW	Total power requirements to fail the soil wedge between the first and second shank,	Pt_{22}
kW	Net power requirements for the second shank**,**	Ptt_2
kW	Power requirements to fail the center wedge for the third shank,	P_{c3}
kW	Power requirements to fail the side circular wedge for the third shank,	P_{s3}
kW	Total power requirements for the third shank,	Pt_3
kW	Power requirements to fail the center wedge between the first and second shank,	P_{c33}
kW	Power requirements to fail the side circular wedge between the second and third shank,	P_{s33}
kW	Total power requirements to fail the soil wedge between the second and third shank,	Pt_{33}
kW	Net power requirements for the third shank,	Ptt_3
kW	Power requirements to fail the center wedge for the fourth shank,	P_{c4}
kW	Power requirements to fail the side circular wedge for the fourth shank,	P_{s4}
kW	Total power requirements for the fourth shank,	Pt_4
kW	Power requirements to fail the center wedge between the third and fourth shank,	P_{c44}
kW	Power requirements to fail the side circular wedge between the third and fourth shank,	P_{s44}
kW	Total power requirements to fail the soil wedge between the third and fourth shank,	Pt_{44}
kW	Net power requirements for the fourth shank,	Ptt_4
kW	Power requirements to fail the disturbed center wedge between the first and second shank when, S= 45cm,	P_{c01}
kW	Power requirements to fail the disturbed side wedge between the first and second shank when, S= 45cm,	P_{s01}
kW	Total power requirements to fail the disturbed wedge between the first and second shank when, S= 45cm,	Ptt_5
kW	Power requirements to fail the disturbed center wedge between the second and third shank when, S= 45cm,	P_{c02}
kW	Power requirements to fail the disturbed side wedge between the second and third shank when, S= 45cm,	P_{s02}
kW	Total power requirements to fail the disturbed wedge between the second and third shank when, S= 45cm,	Ptt_6
kW	Power requirements to fail the disturbed center wedge between the third and fourth shank when, S= 45cm,	P_{c03}
kW	Power requirements to fail the disturbed side wedge between the third and fourth shank when, S= 45cm,	P_{s03}
kW	Total power requirements to fail the disturbed wedge between the third and fourth shank when, S= 45cm,	Ptt_7
kW	Power requirements to pull subsoiler one shank without vibration,	PT_1
kW	Power requirements to pull subsoiler one shank with vibration,	PT_2
kW	Power requirements to pull subsoiler two shanks when, S= 15-30cm without vibration,	PT_3
kW	Power requirements to pull subsoiler two shanks when, S= 45cm without vibration,	PT_4
kW	Power requirements to pull subsoiler two shanks when, S= 15-30cm with vibration,	PT_5
kW	Power requirements to pull subsoiler two shanks when, S= 45cm with vibration,	PT_6
kW	Power requirements to pull subsoiler three shanks when, S= 15-30cm without vibration,	PT_7
kW	Power requirements to pull subsoiler three shanks when, S= 45cm without vibration,	PT_8
kW	Power requirements to pull subsoiler three shanks when, S= 15-30cm with vibration,	PT_9
kW	Power requirements to pull subsoiler three shanks when, S= 45cm with vibration,	PT_{10}

kW	Power requirements to pull subsoiler four shanks when, S= 15-30cm without vibration,	PT_{11}
kW	Power requirements to pull subsoiler four shanks when, S= 45cm without vibration,	PT_{12}
kW	Power requirements to pull subsoiler four shanks when, S= 15-30cm with vibration,	PT_{13}
kW	Power requirements to pull subsoiler four shanks when, S= 45cm with vibration,	PT_{14}

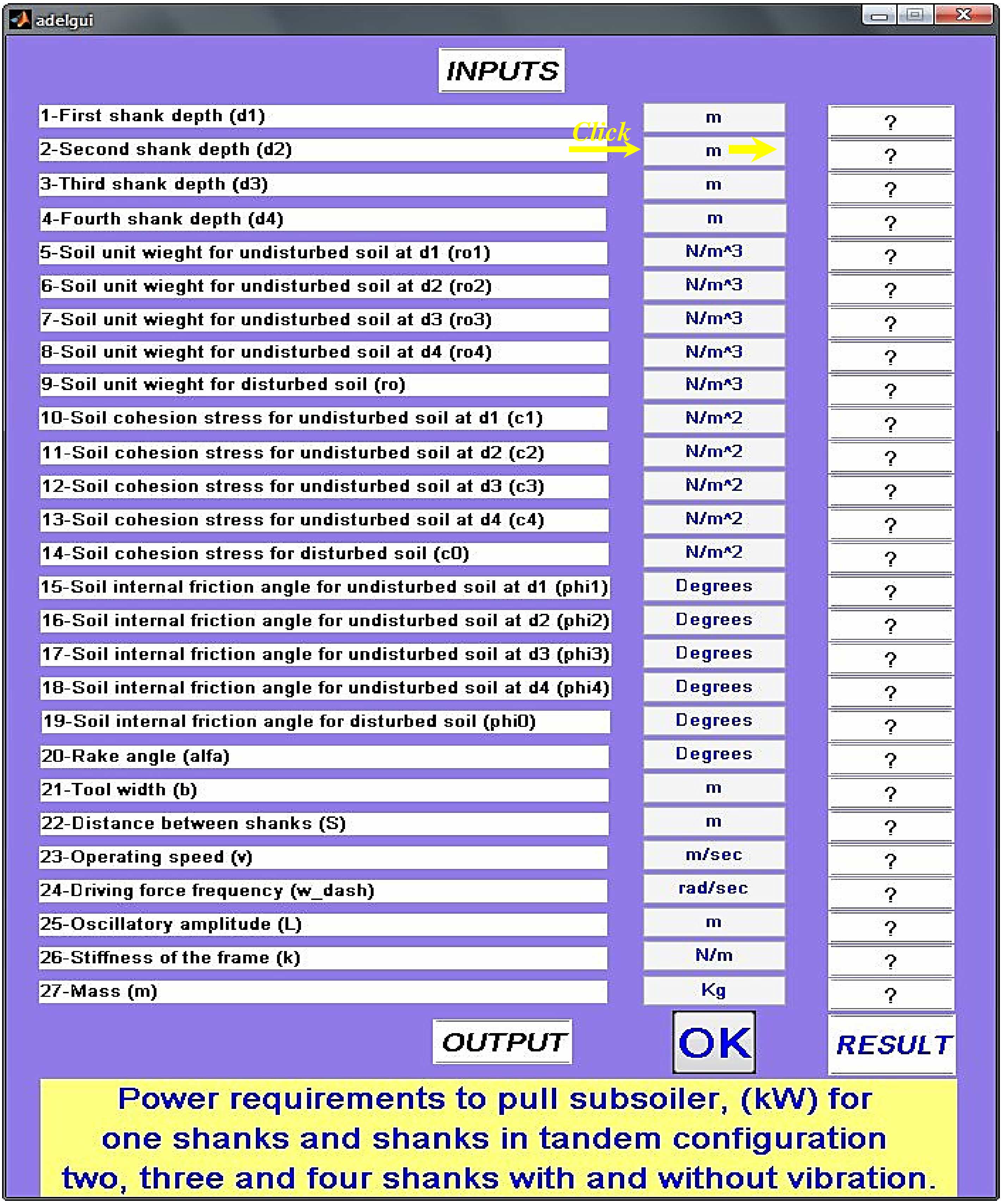

Fig. 9 : Windows of inputs data and their measuring units to predict the power requirements to pull subsoiler as presented in Matlab program.

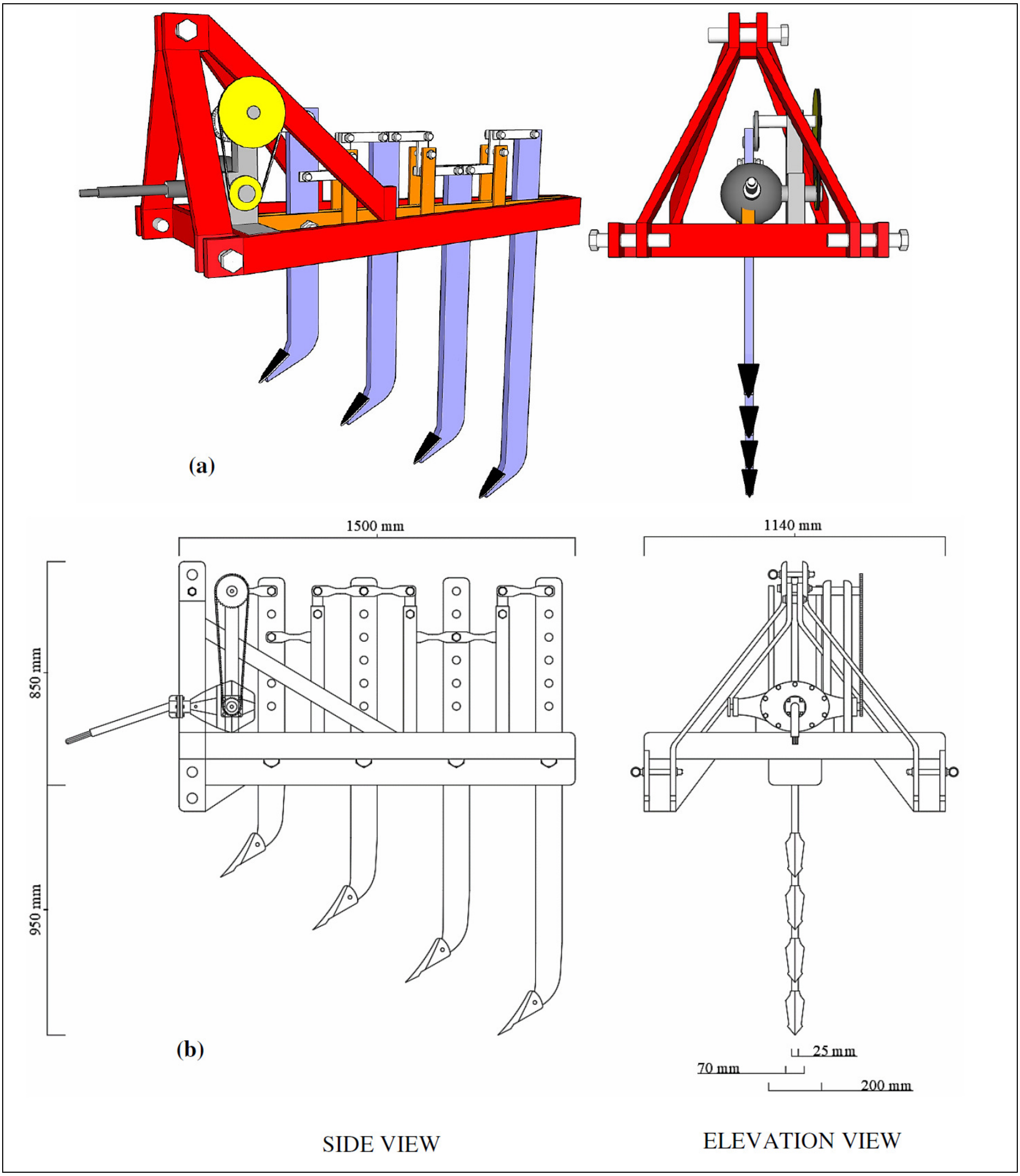

Fig. 10 : Perspective of subsoiler (a) and elevation and side views of it (b).

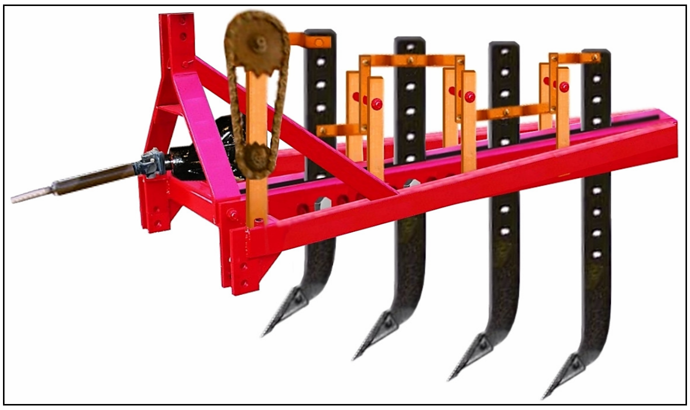

Fig. 11 : Picture of subsoiler.

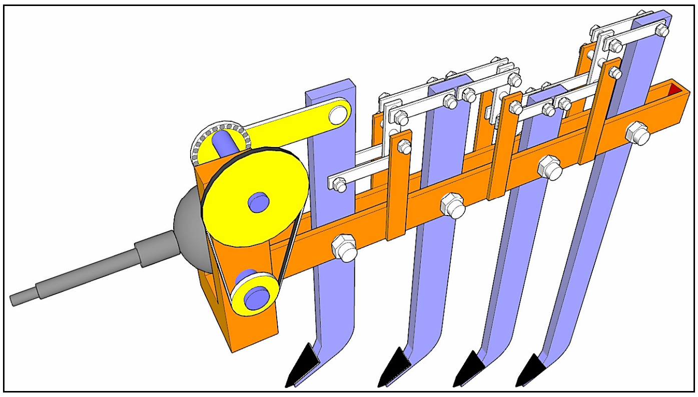

Fig. 12 : Mechanism of vibration system for subsoiler.

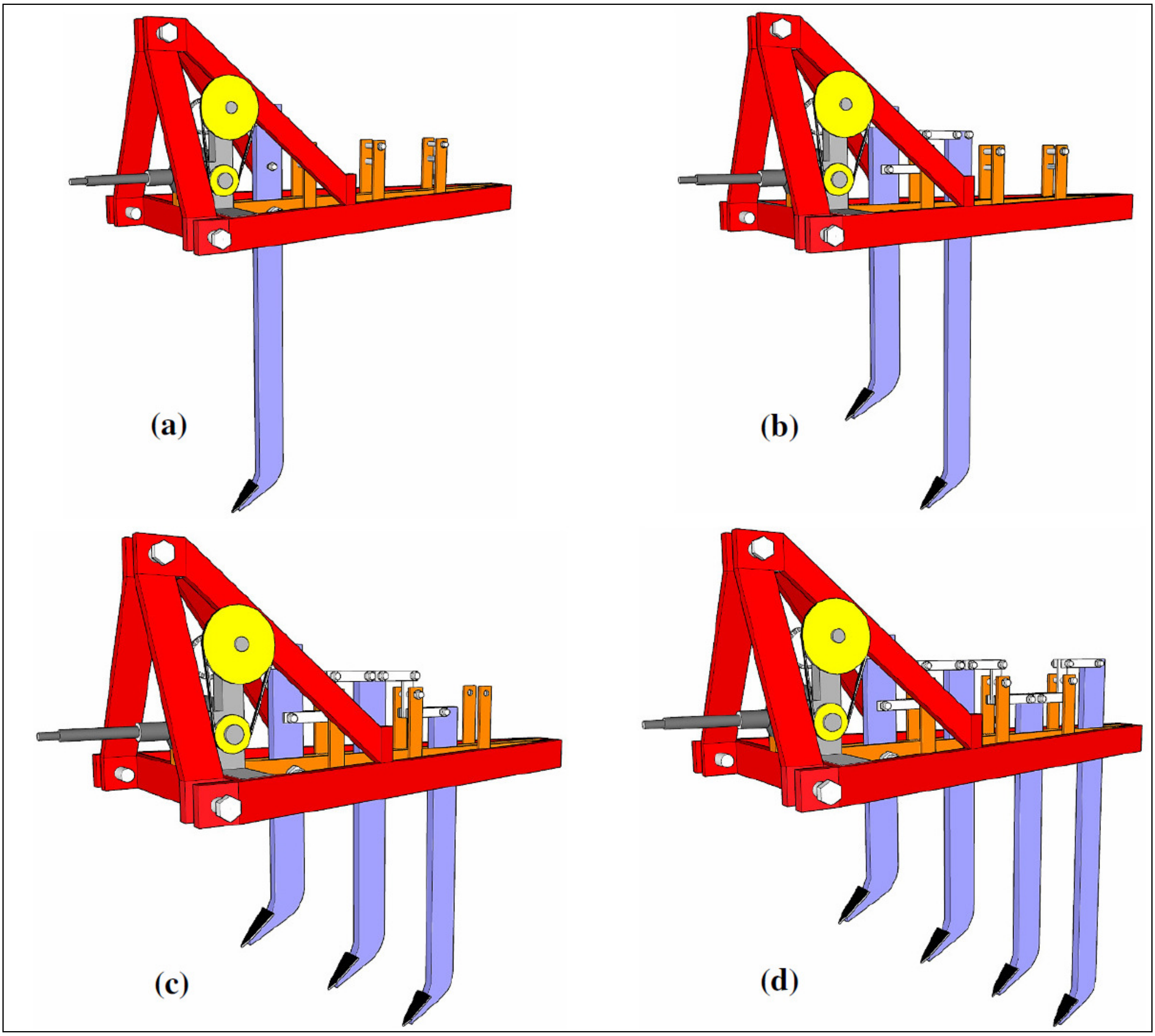

Fig. 13 : Tillage depth division for subsoiler to: (a) one depth 80cm by one shank, (b) two depths 40cm by two shanks, (c) three depths 27cm by three shanks and (d) four depths 20cm by four shanks.

Measurements

Shear ring apparatus

The measurement of soil cohesion and soil internal friction angle was carried out using a shear ring apparatus by (Bekker, 1969), Figure (14). This device consists of a shear ring imbedded in the soil. This enables the operator to apply 105 kPa (15 Ib/in^2) normal pressure comfortably. It also has a calibrated spring, which deflects when the soil is in shear failure, a recording drum on which a recoding pen writes to indicate the values of the shear and normal pressures applied. It also has a handle for the operator to hold and apply the required combination of pressures. The operator forces the circular shear head into the soil so that the shear ring grips the soil and then applies a known amount pressure. Operator twists the handle to apply a shearing force on to the soil. At the point of failure, the shear head starts to turn. The operator takes note this point by making a mark on the recording drum to which is attached a special paper indicating the relationship between normal and shear pressure. Varying amounts of normal pressures are applied and a mark made at each point when the soil starts to fail. A straight line drawn through the points represents the soil failure line or the soil strength. The soil cohesion obtained by interpolating this line so that it cuts the shear stress axis. The intercept is the soil cohesion, C (kPa). The angle between this line and the horizontal gives the value of the soil internal friction angle ϕ.

The shear stress (τ) can be calculate from the torque (T) as: $\tau = \frac{3T}{2\pi(R_2^3 - R_1^3)}$

Where: R_1 and R_2 are the inner and outer radius of the shear ring. Torque is given by the mean load on the tie rods measured as: $T = \frac{(F_1 + F_2)L}{2}$

Where: F_1 and F_2 are forces separated by the distance L.

The normal stress is given by the vertical load N as: $\sigma = \frac{N}{\pi(R_2^2 - R_1^2)}$

From measured shear strength (τ) and normal stress (σ) for soil obtained : $\tau = C + \sigma \tan\phi$.

Where: C = Effective cohesion of the soil (kPa) and ϕ = Internal friction angle of the soil (degrees).

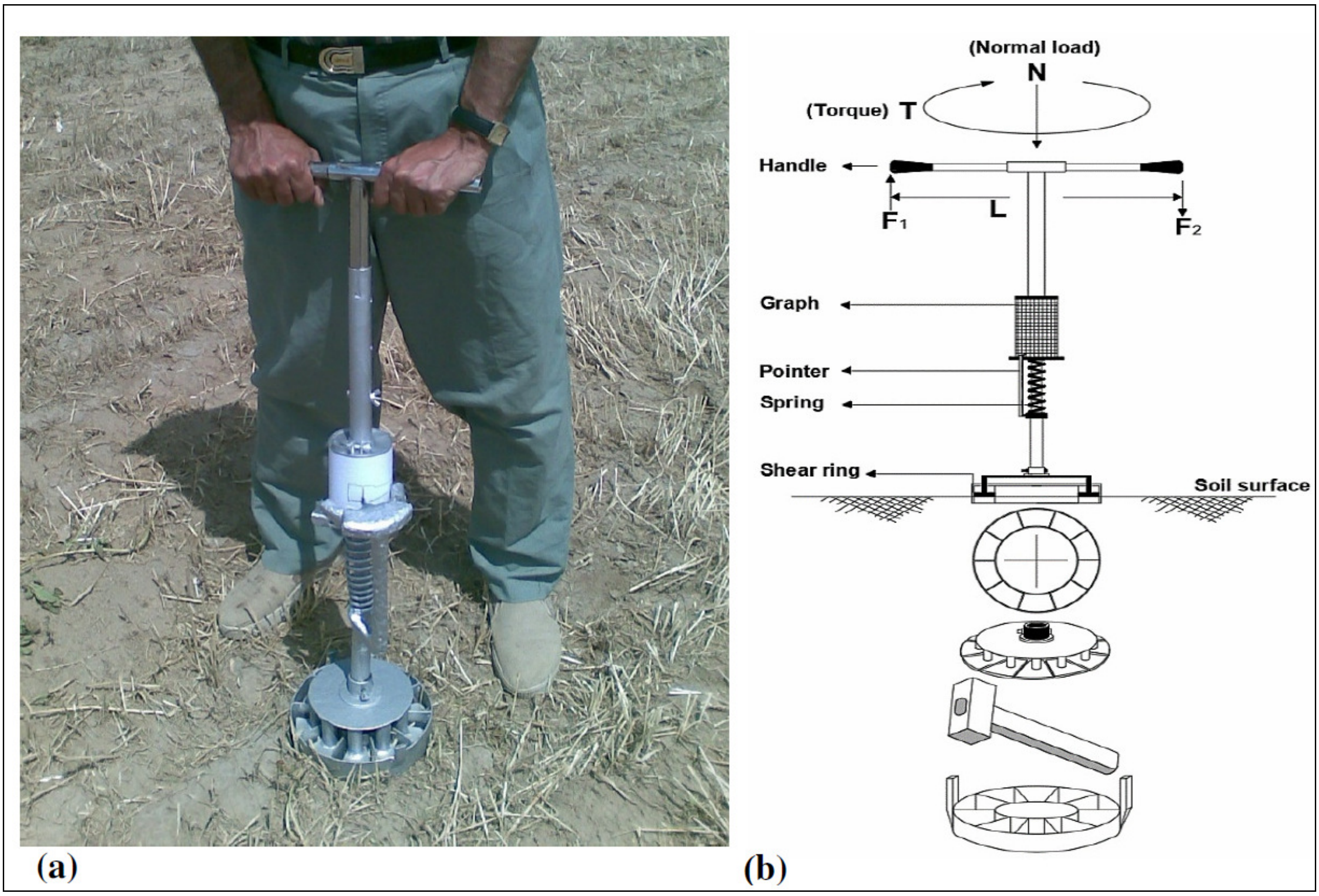

Fig. 14 : Picture of the shear ring in the field (a) and illustration of device parts (b).

Power requirements to pull subsoiler

Pulling force was measured by hydraulic dynamometer, which, coupled between the two tractors with the attaching subsoiler to estimate its draft force. The average of 10 readings of the draft force was taken in 10 seconds intervals. The power requirements to pull subsoiler was estimated from the following equation: P = D x v, where: P = Power requirements to pull subsoiler, kW, D = Draft force, kN and v = Operating speed, m/s.

Fuel consumption rate

Fuel consumption per unit time was determined by measuring the volume of fuel consumed during operation time. It was measured using the fuel meter equipment as shown in Figure (15). The length of line, which marked by the marker tool on the paper sheet represents the fuel consumption. The fuel meter was calibrated prior and the volume of fuel was determined accurately.

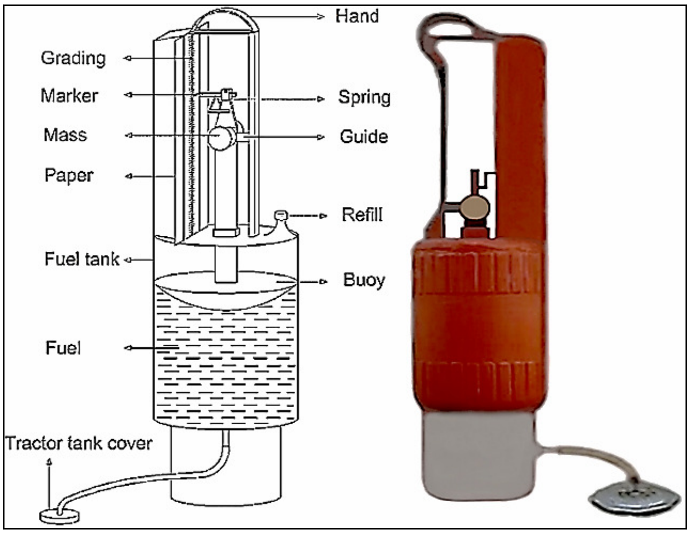

Fig. 15 : Fuel meter for measuring fuel consumption.

- Fuel consumption of the tractor was measured when the tractor carried the subsoiler plow without vibration and the plow raised outside the soil (not plowing) and moving at a constant operating speed (3.6 km / h). = A
- Fuel consumption of the tractor was measured when the tractor carried the subsoiler plow with vibration and the plow raised outside the soil (not plowing) and moving at a constant operating speed (3.6 km / h). = B
- Fuel consumption of the tractor was measured when the tractor carried the subsoiler plow without vibration and the plow was working inside the soil (plowing) and moving at a constant operating speed (3.6 km / h). = C
- Fuel consumption of the tractor was measured when the tractor carried the subsoiler plow with vibration and the plow was working inside the soil (plowing) and moving at a constant operating speed (3.6 km / h). = D

So that:

- Fuel consumption of the tillage operation using the subsoiler plow without vibration = C – A
- Fuel consumption of the tillage operation using the subsoiler plow with vibration only without measured the fuel used to produce the vibration of the plow = D – B

Soil bulk density

Soil bulk density was measured using a core method as described by Black, (1986).

Average infiltration rate

Infiltration characteristics of the studied soil was determined in the field by using a local made double ring (cylinder infiltrometer). The two cylinders were 30 cm deep and formed of steel sheet of 5mm thickness which allow the cylinders to enter the soil with little disturbance. The inner cylinder, from which the infiltration measurements were taken, was 30 cm in diameter. The outer cylinder, which used to form the buffer pond was 60 cm in diameter. The double ring hammered into the soil to a depth of 15 cm. Care was taken to keep the installation depth of the cylinder to be the same in all experiments. Average infiltration rates calculated by Kostiakov equation, (1932):

$$I = 60 * C * T^{m-1},$$

Where: I = Average infiltration rate, (cm/h), c, m = Constants depend on soil properties and initial condition, and T = Time after infiltration started (min).

Results and Discussion

Effect of study treatments on power requirements to pull subsoiler and fuel consumption rate.

Table (2) and Figure (16) represents the effect of tillage depth division, distance between shanks and vibration conditions on power requirements to pull subsoiler (theoretical and measurement) and fuel consumption rate. In general, tillage depth division, distance between shanks and vibration conditions caused a significant decreasing in power requirements to pull subsoiler (theoretical and measurement) and fuel consumption rate. The decreasing percentages for tillage depth division were (27% - 30% and 23%) respectively compared to traditional subsoiler (one shanks at one tillage depth 80cm), for distance between shanks were (26% - 28% - 21%) respectively compared to zero distance (one shank) and for vibration conditions were (23% - 26% - 20%) respectively compared to non-vibrating conditions. The decreasing percentages in theoretical and measurement of power requirements to pull subsoiler and fuel consumption rate for tillage depth division levels (two - three - four) depths were (20% - 27% - 33%), (24% - 30% - 36%) and (19% - 23% - 28%) respectively compared to traditional subsoiler (one shanks at one tillage depth 80cm). Reduced the power requirements for subsoiler as a result of dividing the tillage depth. This can be explained by that, when dividing the tillage depth of 80 cm to four depths of (20 cm) by using four shanks in a tandem configuration, each shank plowing the soil at a depth of (20 cm). Which reduces the soil weight in front of each shank compared to use a one shank at a depth of (80 cm). Similarly, the decreasing percentages in theoretical and measurement of power requirements to pull subsoiler and fuel consumption rate for distance between shanks levels (15cm – 30cm – 45cm) were, (30% - 37% - 13%), (33% - 40% - 17%) and (25% - 31% - 13%) respectively compared to zero distance (one shank). The power requirements of the subsoiler decrease when increasing the distance between shanks from 15 cm to 30 cm and then increase when increasing the distance to (45 cm). This result can be attributable it must be the distance between the tandem shanks is appropriate so that if this distance is 15cm, there will not be enough time to allow the front blade to raise the soil and interference occurs between the blades, which leads to ineffective division of tillage depth and power requirements increase. If the distance between the plows shanks is 45cm, the time will be large between entering the front and rear blades. Which causes the front blade to raise the soil and then drop it (disturbed soil) back to the surface of the soil before entering the rear blade to the soil, which leads to adding loads to the rear blade and increases the power requirements. However, when the distance between the tandem shanks is appropriate (30cm) the power requirements decreased. Because the front blade raises a depth of soil on its surface, so the soil surface exposes in front of the next blade, which raises another depth of soil on it, so the soil surface exposes to the next blade and so on for the following blades. As well as the decreasing percentages in theoretical and measurement of power requirements to pull subsoiler and fuel consumption rate for vibration conditions were, (24% - 28% - 20%) respectively compared to non-vibrating. This result can be attributable the maximum velocity of oscillation is greater than the velocity of the tool carrier which caused decreased in power requirements to pull subsoiler at vibration conditions compared to non-vibration, this is in agreement with Butson and MacIntyre (1981).

Effect of study treatments on soil bulk density and average infiltration rate

Tillage depth division, distance between shanks and vibration conditions caused a significant effects on soil bulk density and average infiltration rate of water as shown in Table (3) and Figure (17). The results showed that all the study treatments caused a good soil disturbed, which appear in the decreasing of soil bulk density and increasing of average infiltration rate of water. The decreasing and increasing percentages of soil bulk density and average infiltration rate of water for tillage depth division were (10% - 25%) compared to traditional subsoiler (one shanks at one tillage depth 80cm), for distance between shanks were (9% - 23%) respectively compared to zero distance (one shank) and for vibration conditions were (5% - 10%) respectively compared to non-vibrating conditions. The decreasing and increasing percentages of soil bulk density and average infiltration rate of water for tillage depth division levels (two - three - four) depths were (6% - 10% - 13%) and (15% - 24% - 36%) respectively compared to traditional subsoiler (one shanks at one tillage depth 80cm). In addition, the decreasing and increasing percentages of soil bulk density and average infiltration rate of water for distance between shanks levels (15cm – 30cm – 45cm) were, (10% - 13% - 4%) and (28% - 35% - 11%) respectively compared to zero distance (one shank). These results can be explained because when dividing the deep tillage depth of the subsoiler into a number of shallow depths, each blade works above the critical depth. Which increases the soil fragmentation, which leads to decrease in the soil bulk density and increase in the average infiltration rate of water, compared to use a single shank at a deep tillage depth under the critical depth. Which reduces the soil fragmentation process and increases soil compaction. Thus, leads to increase in soil bulk density and decrease in the average infiltration rate of water. Also, the decreasing and increasing percentages of soil bulk density and average infiltration rate of water for vibration conditions were, (5% - 10%) respectively compared to non-vibrating, this is in agreement with Gupta and Rajput (1992). The data observed that the decreasing and increasing percentages of soil bulk density and average infiltration rate of water for tillage depth division levels (one - two - three - four) depths were (15% - 20% - 23% - 26%) and (72% - 97% - 114% - 123%) respectively compared before tillage. The decreasing and increasing percentages of soil bulk density and average infiltration rate of water for distance between shanks levels (0cm- 15cm – 30cm – 45cm) were (15% - 24% - 26% - 19%) and (72% - 120% - 131% - 91%) respectively compared before tillage. The decreasing and increasing percentages of soil bulk density and average infiltration rate of vibration conditions were (20% - 24%) and (100% - 120%) respectively compared before tillage.

Table 2 : Effect of study treatments on power requirements to pull subsoiler, kW and fuel consumption rate, L/h.

Fuel consumption rate, L/h	Measurement power requirements, kW	Theoretical power requirements, kW	Vibration of shanks	Distance between shanks, (cm)	Division of soil depth (80cm)
16.8[a]	36.93[a]	38.49[a]	Fixed	0	One depth (80cm), one shank
12.79[e]	25.85[e]	28.63[e]	Vibrated		
13.3[d]	27.28[d]	29.17[e]	Fixed	15	Two depths (40cm), two shanks
10.81[hi]	20.39[hi]	22.74[gh]	Vibrated		
12.01[f]	23.72[f]	26.37[f]	Fixed	30	
9.73[k]	17.42[k]	20.61[i]	Vibrated		
15.09[b]	32.21[b]	35.79[b]	Fixed	45	
11.41[g]	22.04[g]	26.15[f]	Vibrated		
11.54[g]	22.42[g]	25.82[f]	Fixed	15	Three depths (27cm), three shanks
10.27[j]	18.91[j]	20.52[i]	Vibrated		
11.3[g]	21.75[g]	23.48[gh]	Fixed	30	
9.14[l]	15.78[l]	17.18[k]	Vibrated		
14.47[c]	30.51[c]	33.37[c]	Fixed	45	
11.55[g]	22.44[g]	25.87[f]	Vibrated		
11.16[gh]	21.35[gh]	23.94[g]	Fixed	15	Four depths (20cm), four shanks
9.21[l]	15.98[l]	18.78[j]	Vibrated		
10.58[ij]	19.77[ij]	21.71[hi]	Fixed	30	
8.55[m]	14.16[m]	16.83[k]	Vibrated		
14.12[c]	29.52[c]	30.79[d]	Fixed	45	
10.38[ij]	19.21[ij]	22.62[gh]	Vibrated		
0.38283	1.02629	1.35955	L.S.D		

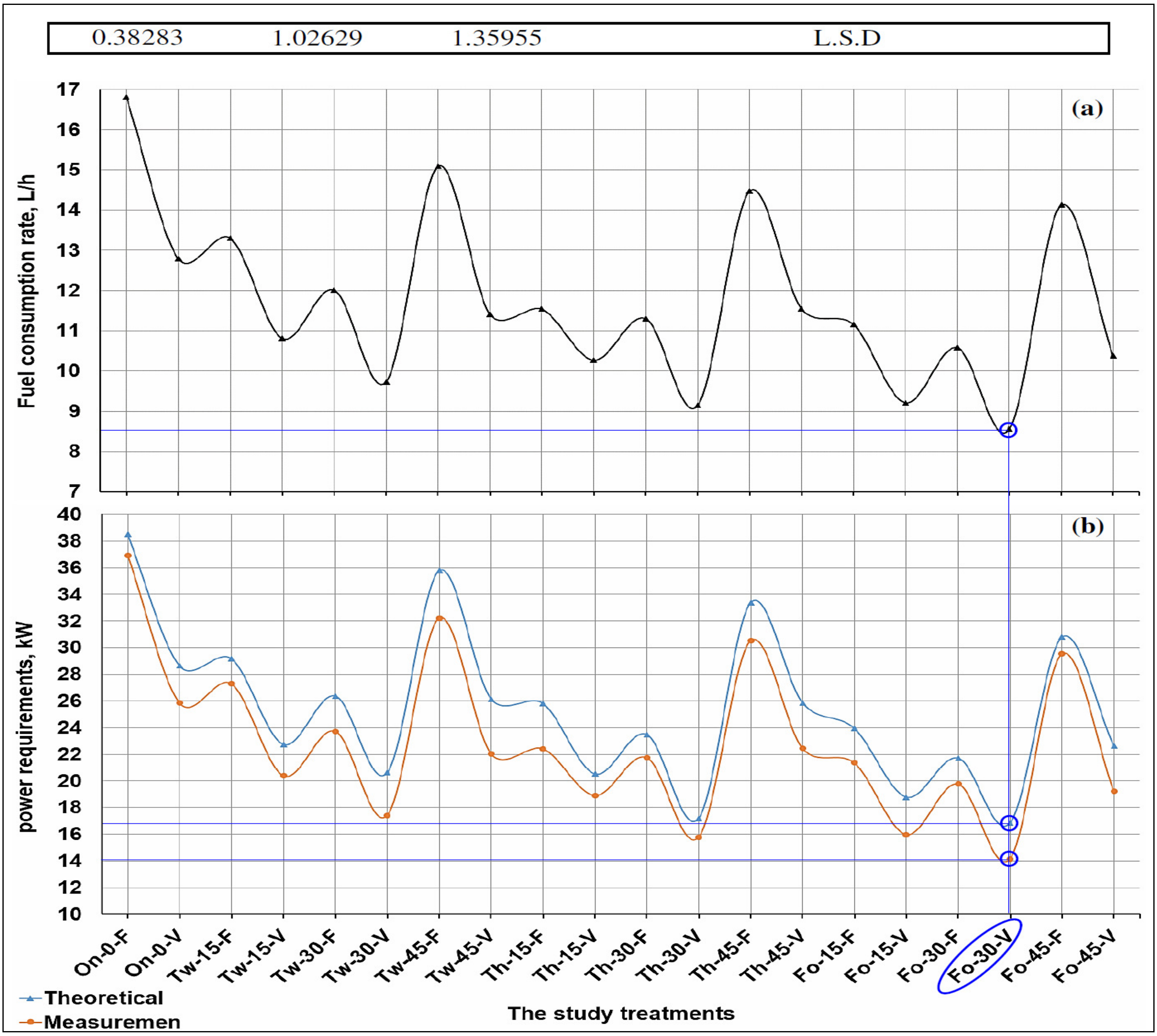

Fig. 16 : Effect of study treatments (On, Tw, Th and Fo = one, two, three and four depths - 0, 15, 30 and 45 = distance between shanks - F and V = fixed and vibrated shanks) on fuel consumption, L/h, (a) and power requirements, kW, (b).

Pearson and partial correlation coefficients for relationship between measurements and study treatments.

Pearson and partial correlation coefficients for relationship between measurements and study treatments illustrated in Table (4). From the statistical analysis of data by SPSS program, the pearson and partial correlation coefficients were derived to illustrate the effect of the study treatments on the soil bulk density, infiltration rate, power requirements and fuel consumption. Where the division of tillage depth and the distance between shanks had, the highest impact on the soil disturbed process and the lowest impact on the power requirements and fuel consumption compared to the vibration treatment. However, the effect of vibration on power requirements and fuel consumption was higher than its impact on soil fragmentation compared to division of tillage depth and the distance between shanks

.

Table 3 : Effect of study treatments on soil bulk density, g/cm^3 and average infiltration rate, L/h

Average infiltration rate after tillage, L/h	**Soil bulk density after tillage, g/cm^3**	**Vibration of shanks**	**Distance between shanks, (cm)**	**Division of soil depth (80cm)**
8.81^{n}	1.41^{a}	Fixed	0	One depth (80cm), one shank
9.82^{l}	1.33^{bc}	Vibrated		
10.31^{k}	1.31^{bcd}	Fixed	15	Two depths (40cm), two shanks
11.34^{h}	1.26^{efg}	Vibrated		
11.11^{i}	1.27^{ef}	Fixed	30	
12.21^{f}	1.21^{ij}	Vibrated		

9.13^{m}	1.39^{a}	Fixed	45	
10.12^{k}	1.32^{bc}	Vibrated		
11.51^{h}	1.25^{fgh}	Fixed	15	Three depths (27cm), three shanks
12.44^{e}	1.2^{ijk}	Vibrated		
11.91^{g}	1.22^{hij}	Fixed	30	
13.23^{c}	1.16^{lm}	Vibrated		
9.61^{l}	1.35^{b}	Fixed	45	
10.91^{ij}	1.28^{def}	Vibrated		
12.63^{e}	1.19^{jkl}	Fixed	15	Four depths (20cm), four shanks
13.61^{b}	1.14^{m}	Vibrated		
12.92^{d}	1.17^{klm}	Fixed	30	
14.12^{a}	1.11^{n}	Vibrated		
10.73^{j}	1.29^{cde}	Fixed	45	
11.75^{g}	1.23^{ghi}	Vibrated		
0.22438	0.02773	L.S.D		
5.43	1.61	Before tillage		

Table 4 : Pearson and partial correlation coefficients for relationship between measurements and study treatments.

Fuel consumption		Measurement power requirements		Average infiltration rate		Soil bulk density		The measurements (dependents) / The treatments (independents)
r	R	r	R	r	R	r	R	
-0.53	-0.32	-0.54	-0.33	0.77	0.61	-0.76	-0.59	Division of soil depth
0.62	0.40	0.63	0.41	-0.70	-0.50	0.68	0.48	Distance between shanks
-0.81	-0.69	-0.82	-0.70	0.63	0.42	-0.62	-0.41	Vibration of shanks

R = Pearson correlation coefficient.
r = Partial correlation coefficient.

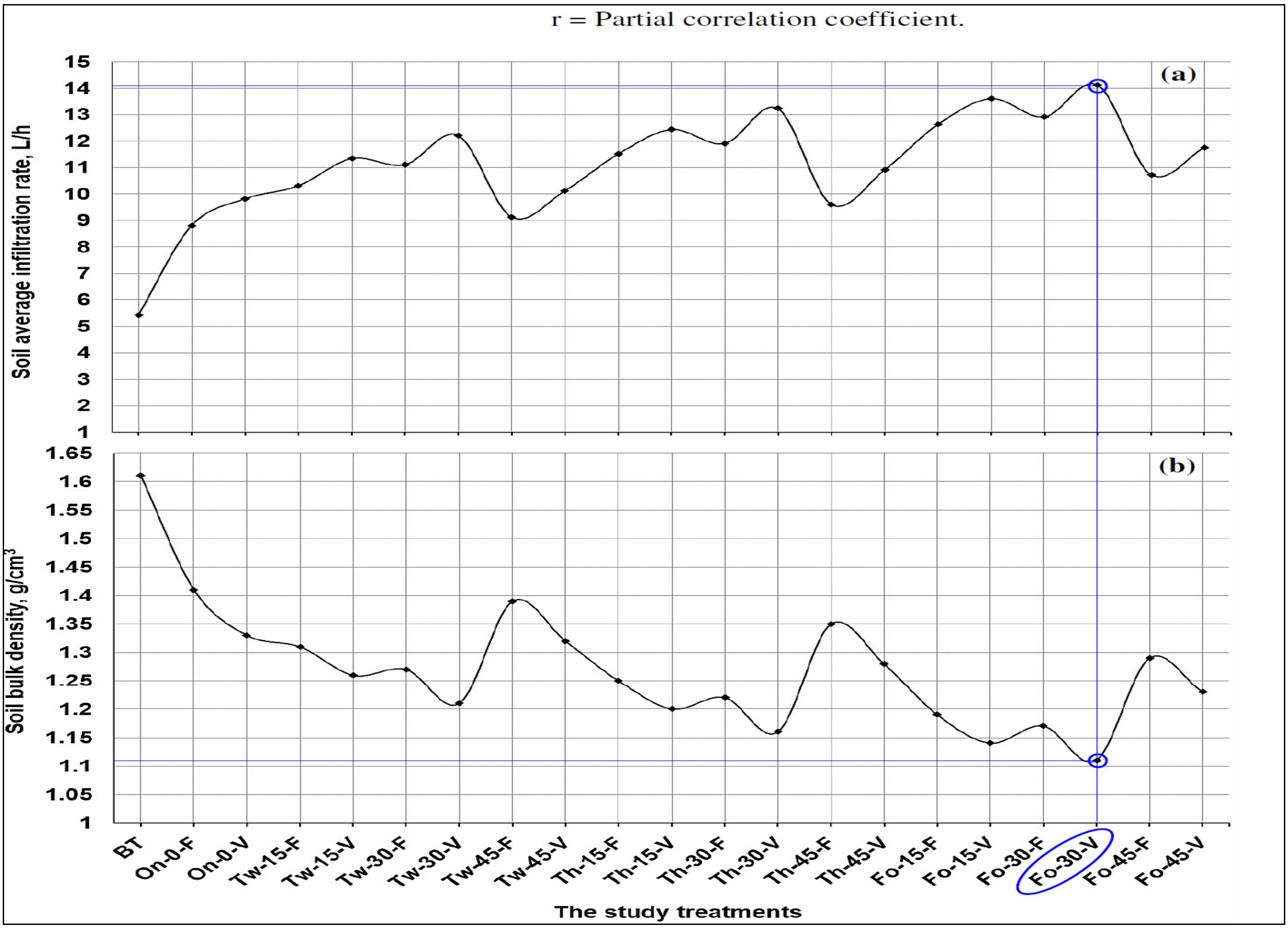

Fig. 17 : Effect of study treatments (On, Tw, Th and Fo = one, two, three and four depths - 0, 15, 30 and 45 = distance between shanks - F and V = fixed and vibrated shanks – BT = before tillage) on average infiltration rate, L/h, (a) and soil bulk density, g/cm^3, (b).

Optimum distance between shanks

In general, the distance between shanks affected on power requirements to pull subsoiler. Where, the power requirements to pull subsoiler decreased when the distance between shanks increased from 15 to 30 cm after that the relationship reversed. Where, the power requirements to pull subsoiler increased at the distance between shanks increased to 45cm for theoretical and measurement power as shown in Figure (18). The curves equations of the relationship between power requirements (theoretical and measurement) and distance between shanks was calculated. Then the equations has been differentiated and equal to zero to obtain the optimum distance between shanks value that achieved the lowest power requirements to pull subsoiler, which was achieved at the distance about 25cm as the following:

For theoretical power:

$$Y = 0.0201\,x^2 - 1.0123\,x + 33.707$$

$$\frac{dy}{dx} = \frac{d}{dx\left(0.0201\,x^2 - 1.0123\,x + 33.707\right)}$$

Equalize the differential result to zero

$$0.0402\,x - 1.0123 = 0$$

$$0.0402\,x = 1.0123$$

$$x = 1.\frac{0123}{0}.0402 = 25.18 \text{ cm}$$

For measurement power:

$$Y = 0.0197\,x^2 - 1.0011\,x + 31.463$$

$$\frac{dy}{dx} = \frac{d}{dx\left(0.0197\,x^2 - 1.0011\,x + 31.463\right)}$$

Equalize the differential result to zero

$$0.0394\,x - 1.0011 = 0$$

$$0.0394\,x = 1.0011$$

$$x = 1.\frac{0011}{0}.0394 = 25.41 \text{ cm}$$

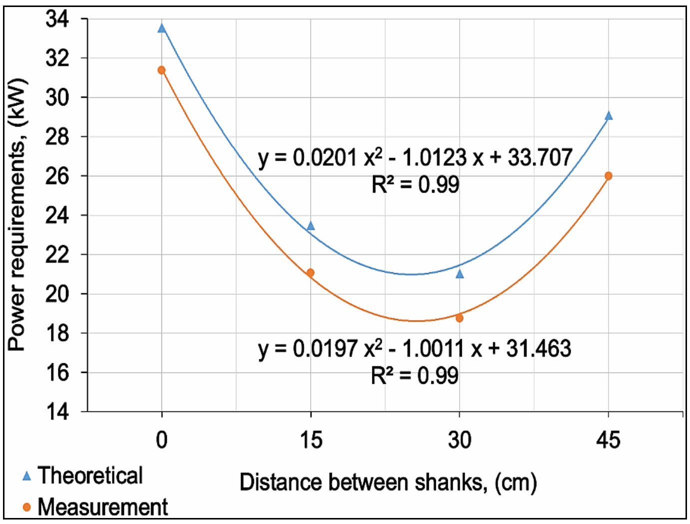

Fig. 18 : optimum distance between shanks at the lowest power requirements to pull subsoiler.

General validation of the mathematical model.

The mathematical model was validated by comparing the theoretically computed values with the experimentally observed values. Measured values were plotted against their predicted values as shown in Figure (19). If there was not discrepancy between the measured data and the predicted results, then all points will lie on a line with a slope of one (the angle with x-axis is equal 45 degree) passing through the origin. For each value of power, the deviation percent was calculated according to the following relationship.

Deviation (%) = ((Measurement power – Theoretical power) / Measurement power)*100.

The prediction error was calculated by dividing the average deviation percent by the number of values. The prediction error was 11.9%. The higher value of the correlation coefficient ($R^2 = 0.94$) indicates that the predicted values are in close agreement with the experimental data.

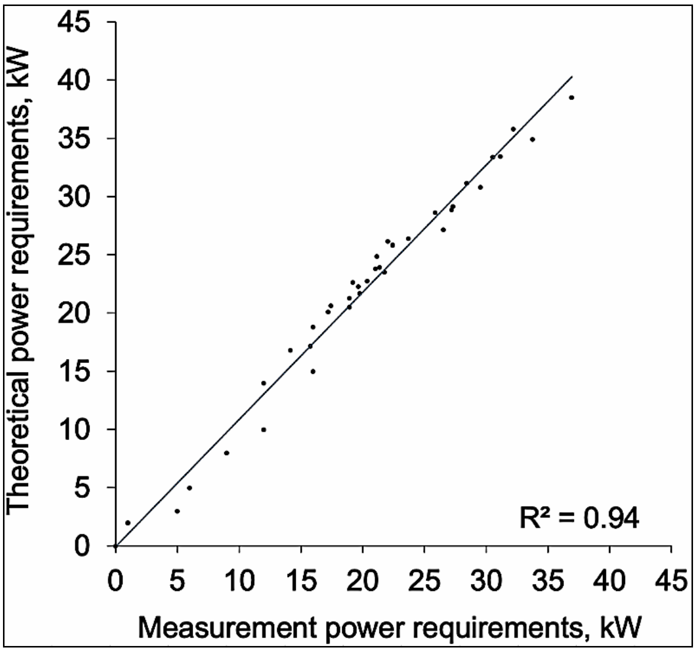

Fig. 19 : Relationship between theoretical and measurement values of the power requirements to pull subsoiler.

Conclusion

The following conclusions can be drawn from the results:

1- The division of tillage depth resulted in a general decrease in the power requirements to pull the subsoiler and the fuel consumption rate, especially when the depth of the tillage was divided into four depths where the percentages of decreasing were (26%-28%), respectively, compared to use one shank at tillage depth 80 cm.

2- The division of tillage depth caused an increase in the performance efficiency of the subsoiler in the soil fragmentation. Where the percentage of decrease in bulk density and increase in the water average infiltration rate water of the soil were (13%-36%), respectively, compared to use one shank at tillage depth 80 cm.

3- Vibration of the subsoiler shanks led to a decrease in the power requirements to pull the plow and the fuel consumption rate about (28% -20%), respectively, compared to non-vibration conditions.

4- Vibration of the subsoiler shanks raising the performance efficiency of the plow in the soil fragmentation. Where the percentage of decrease in bulk density and increase in the water average infiltration rate water of the soil were (5%-10%), respectively, non-vibration conditions.

5- The optimum distance between the subsoiler shanks in tandem configuration that achieved the lowest power consumption was (25 cm).

6- A mathematical model was built on the Matlab program to predict the power requirements to pull the subsoiler under conditions of division of tillage depth and the vibration of the plow shanks, which achieved high accuracy in prediction where $R^2 = 0.94$.

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