Plant Archives Vol. 20, Supplement 2, 2020 pp. 2653-2659 **e-ISSN:2581-6063 (online), ISSN:0972-5210**

DECLINE IN DISSOLVED OXYGEN DUE TO INCREASING TEMPERATURE AND ALGAL BLOOMS: MATHEMATICAL MODEL

Shreya[1] and Preety Kalra[2*]
Department of Mathematics/School of Chemical Engineering and Physical Sciences/Lovely Professional University, Phagwara, Punjab, India.
E-mail: [1]shreyatangri25@gmail.com , [2*] kalra.preety@gmail.com

Abstract

The dissolved oxygen is one of the primary resources required for the survival of the species in an aquatic ecosystem. However, in present times, the dissolved oxygen in water bodies is decreasing due to the increasing carbon dioxide level in the environment as well as the increasing global warming. The effects of these two factors taken together are studied in a mathematical model consisting of variables as average surface temperature, carbon dioxide concentration, density of algal bloom in water and concentration of dissolved oxygen in water. Analytically, stability conditions have been derived and numerically, threshold levels for carbon input have been derived
Keywords : Carbon Dioxide, Global Warming, Algal Blooms, Dissolved oxygen, Stability, Model

Introduction

In recent times, the human activities are leading to a steep rise in the pollution and the carbon dioxide level in the environment. The marine environment is exposed to toxins, toxic chemicals, industrial wastes, etc. by their direct input in water. The presence of pollutants in water results in habitat degradation of aquatic species (He and Wang, 2009). Increased fossil fuel burning, industrial pollutants etc. are some of the factors which lead to rise of carbon emissions in the environment (Shaffer *et al.,* 2009). The increased atmospheric carbon dioxide level contributes to global warming (Solomon *et al.,* 2009). According to Sweetman *et al*. the increase in atmospheric carbon emissions would result in an increase of 1 degree in ocean temperature and a reduction in dissolved oxygen. This will further impact aquatic species ' availability of food and alter their growth and survival rates (Sweetman *et al*., 2017). Climate change also has a direct influence on an ecosystem's structure and functions. The way materials and energy flow in an environment is heavily influenced by climate change (Sarmento *et al*., 2010). Under global warming, the species' metabolic rates are also changed (Danovaro *et al.,* 2001). Studies have also revealed alteration in the growth relationships for aquatic species because of changes in temperature of water (Yvon-Durocher *et al*., 2010). Increased temperatures also lead to melting of ice sheets which proves adverse to the survival of human population (Shukla *et al*., 2017). Additional carbon dioxide level can further result in weak acidification of the water body when occurring in systems with low buffering ability (Barker and Ridgwell, 2012, Raven *et al*., 2005, Stets *et al*., 2017). Poor acidification in the marine environment has the potential to harm reproduction abilities in macrophytes in water and cause alterations in phytoplankton communities (Hasler *et al*., 2018). The rising acidity, contamination and rising water temperatures lead to an increased growth of algal blooms in the water bodies (Chapra *et al*., 2017, Michalak *et al*., 2013, Winter *et al*., 2011). Climate change is also found to have a significant impact on the frequency and abundance of algal blooms affecting the growth and habitat of the algal blooms. The growing algal blooms have negative affect on ecosystem functioning (Glibert *et al*., 2014). They also disturb the aquatic biodiversity and food chains (Riebesell *et al*., 2018). It is seen that, due to pollutants and increased algal growth present in the water bodies, not only the species are affected but the resource i.e. dissolved oxygen is also decreased. The oxygen levels in water fall with rise in algal growth as the algal blooms use up the oxygen in their decomposition process (Bhateria and Jain, 2016, O'Boyle *et al*., 2016). Ocean warming caused by global climate change results in deoxygenation with negative consequences for ocean productivity and marine habitat. Ocean models predict declines of 1 to 7 percent over the next century in the global ocean oxygen stocks, with declines extending into the future for a thousand years or more (Keeling *et al*., 2010). Thus, it is found that global warming is responsible for expansion of hypoxic zones in aquatic bodies as it leads to deoxygenation with reduced global oxygen content and negative effects on ocean productivity and marine habitat (Carstensen *et al*., 2014). The condition of hypoxia due to global warming may also lead to death of various aquatic species dependent on oxygen for their survival (Joos *et al*., 2003). Increased acidity and global warming also lead to the loss of coral reefs ' biodiversity, which poses a threat to their dependent populations (Miller *et al*., 2009, Wolff *et al*., 2018). Therefore, due to the growing acidity and hypoxic conditions, aquatic species face difficulty in their growth and survival (Jansson *et al*., 2015).

The inflating menace of water contamination has been elucidated by various researchers (Kaur *et al*., 2017, Patel *et al*., 2020, Rashid *et al*., 2016, Shukla *et al*., 2019, Singh *et al*., 2018, Singh *et al.,* 2015). Certain researchers have also proposed certain studies to handle the problem of water pollution (Bansal and Geetha, 2018, Bhandari *et al*., 2018, Bhatia *et al*., 2017, Dhanjal *et al*., 2018, Garg *et al.,* 2018, Jain *et al*., 2019, Kandhari and Dutta, 2018, Kanjilal *et al.,* 2014, Karnwal *et al*., 2018, Kaur *et al*., 2018, Kaur and Kamboj, 2019, Kushwaha and Gupta, 2018, Parihar *et al*., 2015, Rahul *et al*., 2018, Sharma *et al*., 2016, Sharma *et al*., 2018, Vise *et al*., 2018). Mathematical modelling has been used a powerful aid in many studies to study biological processes (Kalra and Kumar, 2018, Kumar and Kumar, 2019, Sahoo and Patra, 2020, Yadav and Priyanka, 2019, Yadav, 2019, Kalra and Kumar, 2018). Various mathematical and experimental studies are available which support the decrease of dissolved oxygen due to global warming (Sekerci and Petrovskii, 2018). Mukherjee *et al.* developed a mathematical model for changes in inorganic carbon due to respiration, photosynthesis and deposition of calcium carbonate in aquatic systems (Mukherjee *et al.*, 2002). Also, certain mathematical studies study the effect of pollutants and acidification on aquatic ecosystem individually (Chakraborty *et al.*, 2017, Shukla *et al.*, 2008). But, a combined study which studies the effects of global warming , carbon emissions and algal bloom growth together in one mathematical model is not yet available.

In our study, we have formulated a mathematical model of non-linear differential equations to study the impact of global warming, increased carbon emissions and increased algal bloom growth on the level of dissolved oxygen in water. Threshold levels for maximum amount of carbon input that the aquatic ecosystem can handle has been calculated, above which the carbon dioxide input increase can lead to condition of hypoxia having dreadful effects on aquatic populations. Also, threshold level of oxygen input has also been calculated required to sustain the aquatic life under increased carbon dioxide concentration and global warming. It has been found that under the effects of global warming and carbon dioxide increase in the atmosphere, the dissolved oxygen decreases which can have very harmful effects on the aquatic ecosystem.

Mathematical Model

It is assumed in the proposed mathematical model that due to increase of carbon dioxide concentration in the environment, the atmospheric temperature is increasing which is leading to warming of waters. Also, the water acidity is increasing due to the increasing carbon concentration. These factors are leading to increase in growth of algal blooms in the water bodies. The increasing algal blooms and water temperature further cause decline in the concentration of dissolved oxygen in water. Under these assumptions, let $\tau(t)$ be the average surface temperature, R(t) be the concentration of atmospheric carbon dioxide, $A_g(t)$ be the density of algal blooms and O(t) be the dissolved oxygen concentration in water.

The model is formulated as given below:

$$\frac{d\tau}{dt} = \phi(R - R_0) - \delta(\tau - \tau_0) \quad (1)$$

$$\frac{dR}{dt} = Q - \alpha R - \frac{hRA_g}{1+aR} \quad (2)$$

$$\frac{dA_g}{dt} = \frac{hRA_g}{1+aR} - \beta A_g \quad (3)$$

$$\frac{dO}{dt} = I - dA_gO - nO - d_3O\tau \quad (4)$$

with initial conditions

$\tau(0) \geq \tau_0, R(0) \geq R_0, A_g(0) \geq 0, O(0) \geq 0.$

The model parameters are as given below:

ϕ is the growth rate coefficient corresponding to average surface temperature, δ is the coefficient of depletion of average surface temperature. R_0 is the level of carbon dioxide in absence of pollution and human activities. τ_0 is the average surface temperature in absence of rising carbon dioxide levels. Q is the rate of increase of carbon dioxide due to human activities. α is the natural depletion rate of carbon concentration. h gives the growth rate of algal blooms due to increasing carbon concentration in water and rising water acidity and a represents the proportionality constant. β is the natural death rate of algal blooms. I is the input rate of oxygen in water. d gives the depletion rate of dissolved oxygen due to decomposition process of algal blooms in water. n is the natural depletion rate of oxygen and d_3 gives the rate of decrease of dissolved oxygen concentration due to low solubility of oxygen in water due to global warming.

Dynamical Behaviour

In order to carry out the analysis of the model given by equations (1) - (4), the dynamical behaviour of the mathematical model shall be studied in this section.

1. Boundedness and Positive Invariance

In order to show the boundedness of the solutions of the model given by equations (1)-(4), the following lemma shall be proved.

Lemma 3.1 All the solutions of the system of equations given by (1)-(4) lie in the region

$$\varpi_1 = \left\{ \begin{matrix} (\tau, R, A_g, O) \in R_+^4 : \tau_0 \leq \tau \leq \tau_M, R_0 \leq R \leq R_M, \\ 0 < O < O_M, 0 < R + A_g + O \leq W_{1M} \end{matrix} \right\},$$

as $t \to \infty$, for all initial values $\tau(0), R(0), A_g(0), O(0)$ as given for the model consisting of equations (1)-(4) where, $\tau_M = \frac{\phi R_M + \delta\tau_0}{\delta}, R_M = \frac{Q}{\alpha}, O_M = \frac{I}{n}, W_{1M} = \frac{Q + I}{b_{11}}, b_{11} = \min(\alpha, \beta, n)$

Proof: From equation (2) we get,,

$$\frac{dR}{dt} \leq Q - \alpha R$$

then by usual comparison theorem, as $t \to \infty$,

$$\limsup_{t\to\infty}(R, t) \leq \frac{Q}{\alpha} = R_M$$

From equation (1) we get,

$$\frac{d\tau}{dt} + \delta\tau \leq \phi R_M + \delta\tau_0$$

By comparison theorem we get as $t \to \infty$,

$$\limsup_{t\to\infty}(\tau, t) \leq \frac{\phi R_M + \delta\tau_0}{\delta} = \tau_M$$

From equations (2)-(4),

$$\frac{d(R + A_g + O)}{dt} \leq Q + I - \alpha R - \beta A_g - nO$$

then by comparison theorem we get as $t \to \infty$,

$$\limsup_{t\to\infty}(R + A_g + O, t) \leq \frac{Q + I}{b_{11}} = W_{1M}$$

where b_{11}= min(α, β, n).

From equation (4) ,

$$\frac{dO}{dt} \leq I - nO$$

By comparison theorem we get as $t \to \infty$,

$$\limsup_{t\to\infty}(O, t) \leq \frac{I}{n} = O_M$$

Theorem 3.2: The box ϖ_1 in the space (τ, R, A_g, O) is compact and positive invariant.

Proof: Consider a box ϖ_1 in space (τ, R, A_g, O). One vertex of the box is considered to be at origin and the other vertex at V= $(\tau', R', A_g{}', O')$. Also, consider $\tau' > \tau, R' > R, A_g' > A_g$ and O' > O. For calculating the angle of flow that is made with each face of ϖ_1 not lying in coordinate planes, let ρ_1, ρ_2, ρ_3 and ρ_4 be the outward unit normal vectors to planes $G_1: \tau = \tau', G_2: R = R', G_3 = A_g = A_g'$ and $G_4: O = O'$.

We get from equation (4),

$$\rho_4 \frac{dV}{dt} | G_4 \leq I - nO' - d_3 O' \tau_0$$

since $O' \leq \frac{I}{n}$

$$\rho_4 \frac{dV}{dt} | G_4 \leq -d_3 O' \tau_0$$

hence $\rho_4 \frac{dV}{dt} | G_4 \leq 0$

Similarly, $\rho_1 \frac{dV}{dt} \Big| G_1 \leq 0, \ \rho_2 \frac{dV}{dt} \Big| G_2 \leq 0$,

$$\rho_3 \frac{dV}{dt} | G_3 \leq 0.$$

Hence, ϖ_1 is the region of attraction for model given by equations (1)-(4).

2. Equilibrium points

In this sub-section, the feasible equilibria of the system given by equations (1) - (4) shall be calculated.

1. Boundary equilibrium $E_1(\hat{\tau}, 0, 0, \hat{O})$, where $\hat{R} = 0, \hat{A}_g = 0$ i.e. the carbon dioxide emissions and the algal population density is taken as zero.

$$\hat{\tau} = \frac{\delta \tau_0 - \phi R_0}{\delta}, \hat{\tau} > 0, \text{when } \ \delta \tau_0 - \phi R_0 > 0 \quad (5)$$

$$\hat{O} = \frac{I}{n + d_3 \hat{\tau}} \quad (6)$$

$$\hat{O} > 0 \text{ if } n + d_3 \hat{\tau} > 0x. \quad (7)$$

2. Interior equilibrium $E_2(\tau^*, R^*, A_g^*, O^*)$, where ,

$$R^* = \frac{\beta}{h - a\beta} \quad (8)$$

$$R^* > 0 \quad \text{if} \quad h > a\beta \quad (9)$$

$$\tau^* = \frac{\phi(R^* - R_0) + \delta \tau_0}{\delta} \quad (10)$$

$$\tau^* > 0 \quad \text{as} \quad R^* \geq R_0$$

$$A_g^* = \frac{(Q - \alpha R^*)(1 + aR^*)}{hR^*} \quad (11)$$

$$A_g^* > 0 \quad \text{if} \quad Q > \alpha R^* \quad (12)$$

$$O^* = \frac{I}{dA_g^* + n + d_3 \tau^*} \quad (13)$$

As $\tau^* > 0$, hence $O^* > 0$.

3. Local Stability

In this sub-section we shall carry out the local stability analysis for the model given by equations (1) - (4).

1. For the variational matrix associated with the boundary equilibrium E_1, the characteristic equation is given by

$$\left(-\frac{I}{\hat{O}} - \lambda\right)(-\delta - \lambda)(-\alpha - \lambda)(-\beta - \lambda) = 0 \quad (14)$$

$$\lambda_1 = -\frac{I}{\hat{O}}, \quad \lambda_2 = -\delta, \quad \lambda_3 = -\alpha, \quad \lambda_4 = -\beta \quad (15)$$

The nature of roots of the equation (14) shows that the equilibrium point E_1 is asymptotically stable.

2. For the variational matrix associated with the interior equilibrium E_2, the characteristic equation is given by

$$\left(-\frac{I}{O^*} - \lambda\right)(-\delta - \lambda)(\lambda^2 - Z_1 \lambda + Z_2 Z_3) = 0 \quad (16)$$

where

$$Z_1 = -\alpha - hA_g^* + \frac{haA_g^* R^*}{(1 + aR^*)^2}, \quad Z_2 = \frac{hR^*}{1 + aR^*}, \quad Z_3 = \frac{hA_g^*}{1 + aR^*} - \frac{haA_g^* R^*}{(1 + aR^*)^2}$$

$$\lambda_1 = -\frac{I}{O^*}, \quad \lambda_2 = -\delta \quad (17)$$

Using Routh's criteria, the equilibrium point E_2 is stable only if the following is satisfied,

$$\alpha + hA_g^* > \frac{ha A_g^* R^*}{(1 + aR^*)^2} \quad (18)$$

$$\frac{aR^*}{1 + aR^*} < 1 \quad (19)$$

4. Global Stability

In this sub-section,for the model given by equations (1) - (4), we shall study the global stability behaviour about the interior equilibrium E_2.

Theorem 4.1 The following inequalities should hold for the model given by the system of equations (1)-(4) to be globally stable.

$$\delta\left(\alpha + \frac{hA_g^*}{(1 + aR^*)(1 + aR)}\right) > \phi^2,$$

$$\left(\alpha + \frac{hA_g^*}{(1 + aR^*)(1 + aR)}\right)\left(\beta - \frac{hR}{1 + aR}\right) >$$

$$\left(\frac{hR}{1 + aR} - \frac{hA_g^*}{(1 + aR^*)(1 + aR)}\right)^2,$$

$$\left(\beta - \frac{hR}{1 + aR}\right)(dA_g + n + d_3 \tau) > (dO^*)^2,$$

$$\delta(dA_g + n + d_3 \tau) > (d_3 O^*)^2.$$

Proof: Consider a positive definite function,

$$Z = \frac{1}{2}(\tau - \tau^*)^2 + \frac{1}{2}(R - R^*)^2 + \frac{1}{2}(A_g - A_g^*)^2 + \frac{1}{2}(O - O^*)^2.$$

Differentiating w.r.t. t we get,

$$\frac{dZ}{dt} = -[b_{11}(\tau - \tau^*)^2 + b_{22}(R - R^*)^2 + b_{33}(A_g - A_g^*)^2 + b_{44}(O - O^*)^2 + b_{12}(\tau - \tau^*)(R - R^*) + b_{14}(\tau - \tau^*)(O - O^*) + b_{23}(R - R^*)(A_g - A_g^*) + b_{34}(A_g - A_g^*)(O - O^*)] \quad (20)$$

where,

$$b_{11} = \delta, \quad b_{22} = \alpha + \frac{hA_g^*}{(1 + aR^*)(1 + aR)},$$

$$b_{33} = \beta - \frac{hR}{1 + aR}, \quad b_{14} = d_3 O^*, \quad b_{34} = dO^*,$$

$$b_{44} = dA_g + n + d_3 \tau, \quad b_{12} = -\phi,$$

$$b_{23} = \frac{hR}{1 + aR} - \frac{hA_g^*}{(1 + aR^*)(1 + aR)}.$$

Using Sylvester's criteria we get,

$$b_{11} b_{22} > b_{12}{}^2, \quad b_{22} b_{33} > b_{23}{}^2, \quad b_{33} b_{44} > b_{34}{}^2,$$

$$b_{11} b_{44} > b_{14}{}^2$$

Hence,the system of equations (1)-(4) shall be globally stable if the following are satisfied,

$$\delta\left(\alpha+\frac{hA_g^*}{(1+aR^*)(1+aR)}\right)>\phi^2, \quad (21)$$

$$\left(\alpha+\frac{hA_g^*}{(1+aR^*)(1+aR)}\right)\left(\beta-\frac{hR}{1+aR}\right)>$$

$$\left(\frac{hR}{1+aR}-\frac{hA_g^*}{(1+aR^*)(1+aR)}\right)^2 \quad (22)$$

$$\left(\beta-\frac{hR}{1+aR}\right)(dA_g+n+d_3\tau)>(dO^*)^2, \quad (23)$$

$$\delta(dA_g+n+d_3\tau)>(d_3O^*)^2. \quad (24)$$

Numerical Example

For the equations given in (1) - (4), we shall consider the values of parameters as given below:

$$\phi=0.03, \delta=0.3, R_0=5.5, \tau_0=13.5, \alpha=0.04,$$

$$h=0.4, a=0.7, \beta=0.55, I=23.44,$$

$$d=0.6, n=0.3, Q=6.358, d_3=0.001.$$

With these values of parameters, the following value of variables at interior eqilibrium point $E_2(\tau^*,R^*,A_g^*,O^*)$ are obtained, $\tau^*=16.6167$, $R^*=36.6667$, $A_g^*=8.8933$, $O^*=6.6193$.

For the above mentioned set of values, the conditions of stability given by equations (18) - (19), (21) - (24) are satisfied. Therefore, the interior equilibrium E_2 is stable. The same is supported by figure (1). Also, the dissolved oxygen equilibrium value is similar to the threshold level of dissolved oxygen concentration in fresh water bodies (Nürnberg, 2002).

Conclusion

From the stability analysis of the model given by equations (1) - (4), it is concluded that the existence, boundedness and stability conditions at interior equilibrium E_2 given by equations (9),(12), (18) - (19), (21) - (24) are satisfied. E_2 is found to be locally stable as shown in figure (1). Also, the interior equilibrium point is globally stable as shown in figure (2). The stability analysis shows that dissolved oxygen concentration decreases with rise in global warming as shown in figure (3). This is also supported by equation (13). Also, with the increase in carbon dioxide concentration, the dissolved oxygen level decreases as supported by equation (13) and shown in figure (4). It is also observed that if the value of rate of increase of carbon dioxide increases more than Q=6.694, value of dissolved oxygen drops below 2.00. Thus, a condition of hypoxia arises which can be detrimental for the aquatic population. Also, at value of carbon dioxide increase rate Q=6.83833, the dissolved oxygen level decreases to zero which can lead to fish kills. Hence, we get a threshold level for rate of increase of carbon, above which the survival of species in an aquatic ecosystem is not possible. It is also shown that Q=6.83833, if the value of dissolved oxygen input is maintained above I= 24.115, the dissolved oxygen level again starts to increase. This is shown in figure (5). Thus, for the survival of aquatic species under the carbon dioxide increase rate (Q) greater than 6.83833, the dissolved oxygen input in water has to be maintained above I=24.115. Thus, it is concluded from our study that the rising global warming and carbon dioxide levels in atmosphere greatly decrease the dissolved oxygen level in water, thus, harming the aquatic ecosystem.

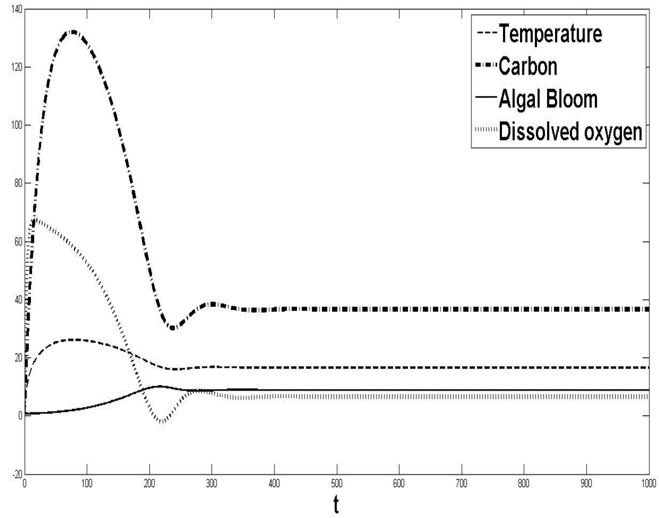

Fig. 1: Local stability behaviour at interior equilibrium

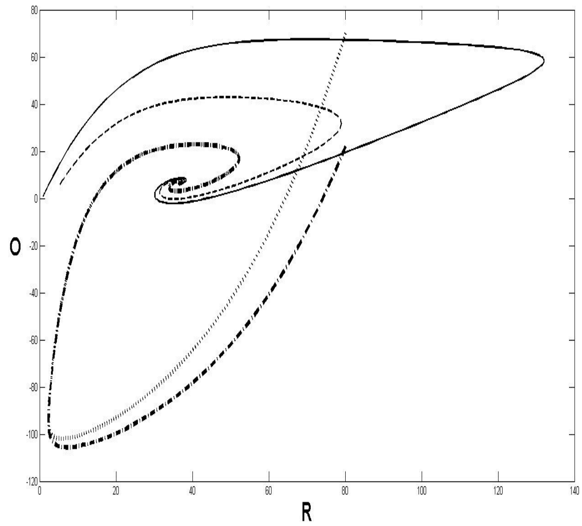

Fig. 2: Global stability behaviour at interior equilibrium point

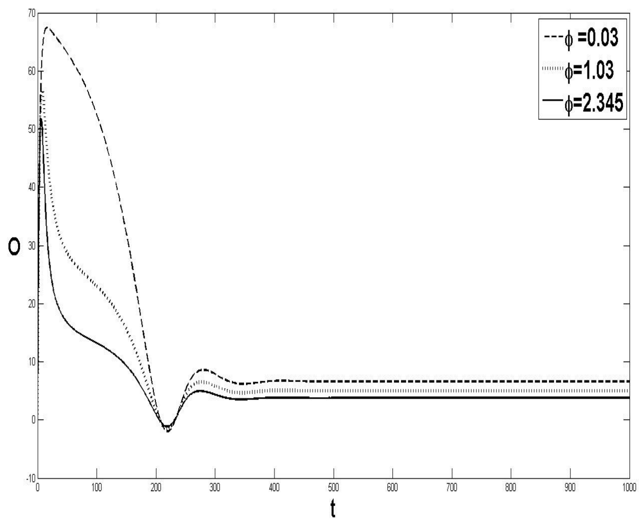

Fig. 3: Decrease in dissolved oxygen with increasing global warming

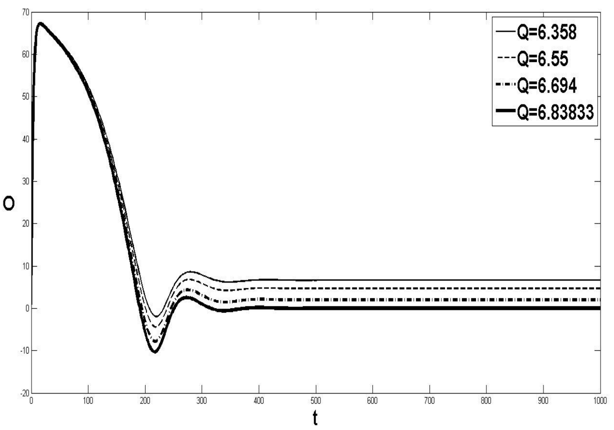

Fig. 4: Decrease in dissolved oxygen with increasing carbon dioxide

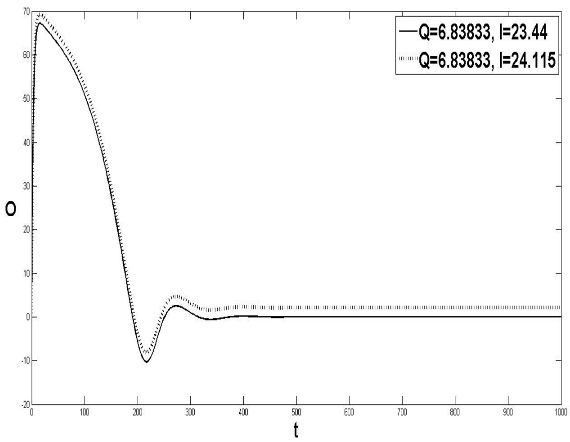

Fig. 5: Graph for threshold value of dissolved oxygen

References

Bansal, S. and G. Geetha (2018). A portable and low cost multi-sensor for real time remote sensing of water quality in agriculture. Pertanika J. Sci. Technol., 26: 1199 - 1212.

Barker, S. and A. Ridgwell (2012). Ocean acidification. Nat. Educ. Knowl., 3: 21.

Bhandari, V., J. Singh and A. Singh (2018). Application of smart polymer hyrogel to cope environmental and health care issues. Poll. Res., 37: 171-176.

Bhateria, R. and D. Jain (2016). Water quality assessment of lake water: a review. Sustain. Water Resour. Manag., 2: 161-173.

Bhatia, D., N.R. Sharma, J. Singh and R.S. Kanwar (2017). Biological methods for textile dye removal from wastewater: A review. Crit. Rev. Env. Sci. Tech., 47: 1836-1876.

Carstensen, J., J. H. Andersen, B. G. Gustafsson and D. J. Conley (2014). Deoxygenation of the Baltic Sea during the last century. Proc. Natl. Acad. Sci., 111: 5628-5633.

Chakraborty, S., P.K. Tiwari, S. K. Sasmal, A. K. Misra and J. Chattopadhyay (2017). Effects of fertilizers used in agricultural fields on algal blooms. Eur. Phys. J-Spec. Top., 226: 2119-2133.

Chapra, S. C., B. Boehlert, C. Fant, V. J. Bierman Jr, J. Henderson, D. Mills, D. M. Mas, L. Rennels, L. Jantarasami, J. Martinich and K.M. Strzepek (2017). Climate change impacts on harmful algal blooms in US freshwaters: a screening-level assessment. Environ. Sci. Technol., 51: 8933-8943.

Danovaro, R., A. Dell'Anno, M. Fabiano, A. Pusceddu and A. Tselepides (2001). Deep-sea ecosystem response to climate changes: the eastern Mediterranean case study. Trends Ecol. Evol., 16: 505-510.

Dhanjal, D. S., S. Singh, D. Bhatia, J. Singh, N. R. Sharma and R. S. Kanwar (2018). Pre-treatment of the municipal wastewater with chemical coagulants. Poll. Res., 37: 32-38.

Garg, M., R. Gupta, S. Prasher, N. R. Sharma and P. Constabel (2018). Integrating natural and engineered remediation strategies for water quality management within a low-impact development (LID) approach. Environ. Sci. Pollut. Res., 25: 29304-29313.

Glibert, P. M., J. I. Allen, Y. Artioli, A. Beusen, L. Bouwman, J. Harle, R. Holmes and J. Holt (2014). Vulnerability of coastal ecosystems to changes in harmful algal bloom distribution in response to climate change: projections based on model analysis. Glob. Chang. Biol., 20: 3845-3858.

Hasler, C. T., J. D. Jeffrey, E. V. Schneider, K. D. Hannan, J.A. Tix and C. D. Suski (2018). Biological consequences of weak acidification caused by elevated carbon dioxide in freshwater ecosystems. Hydrobiologia, 806: 1-12.

He, J. and K. Wang (2009). The survival analysis for a population in a polluted environment. Nonlinear Anal. Real World Appl., 10: 1555-1571.

Jain, P., A. Andotra, A. Aziz, P. Kaur, A. Mahajan and A. Kumar (2019). Phytoremediation- a miracle technique for waste water treatment. Res. J. Pharm. Technol., 12: 2009-2016

Jansson, A., J. Norkko, S. Dupont and A. Norkko (2015). Growth and survival in a changing environment: Combined effects of moderate hypoxia and low pH on juvenile bivalve Macoma balthica. J. Sea Res., 102: 41-47.

Joos, F., G. K. Plattner, T. F. Stocker, A. Körtzinger and D. W. Wallace (2003). Trends in marine dissolved oxygen: Implications for ocean circulation changes and the carbon budget. Eos Trans. AGU, 84: 197-201.

Kalra, P. and P. Kumar (2018). The Study of Effect of Toxic Metal on Plant Growth Dynamics with Time Lag: A Two-Compartment Model. J. Math. Fundam. Sci. , 50: 233-256.

Kalra, P. and P. Kumar (2018). The study of time lag on plant growth under the effect of toxic metal: A mathematical model. Pertanika J. Sci. Technol., 26: 1131 - 1154.

Kandhari, D. and J. Dutta (2018). Efficacy of almond shells for removal of dye from waste water. Poll. Res., 37: 25-31.

Kanjilal, T., S. Babu, K. Biswas, C. Bhattacharjee and S. Datta (2014). Application of mango seed integuments as bio-adsorbent in lead removal from industrial effluent. Desalin. Water Treat., 56: 984-996.

Karnwal, A., V. Bhardwaj, A. Dohroo, A. K. Upadhyay and R. Bala (2018). Effect of microbial surfactants on heavy metal polluted wastewater. Poll. Res., 37: 39-46.

Kaur, H. and V. Kamboj (2019). Bioremediation of textile waste water by plant ash. Foods Raw Mater.,7: 240-246.

Kaur, H., M. Girdhar and A. Mohan (2018). Acid mine drainage: An introduction and treatment strategies. Poll. Res., 37: 83-91.

Kaur, N., R. Gupta and A. Saini (2017). Effect of heavy metal toxicity on commercially important fishes in Harike wetland - A review. Indian J. Environ. Prot., 37:779-785.

Keeling, R. F., A. Körtzinger and N. Gruber (2010). Ocean deoxygenation in a warming world. Annu. Rev. Mar. Sci., 2:199-229.

Kumar, A. and P. Kumar (2019). Application of Markov process/mathematical modelling in analysing communication system reliability. Int. J. Qual.Reliab. Manage., 37:354-371.

Kushwaha, K. and J. Gupta (2018). Vermiremidiation in wastewater management. Poll. Res., 37: 64-70.

Michalak, A. M., E. J. Anderson, D. Beletsky, S. Boland, N. S. Bosch, T. B. Bridgeman, J. D. Chaffin, K. Cho, R. Confesor, I. Daloğlu and J.V. DePinto (2013). Record-setting algal bloom in Lake Erie caused by agricultural and meteorological trends consistent with expected future conditions. Proc. Natl. Acad. Sci., 110: 6448-6452.

Miller, J., E. Muller, C. Rogers, R. Waara, A. Atkinson, K. R. T. Whelan, M. Patterson and B. Witcher (2009). Coral disease following massive bleaching in 2005 causes 60% decline in coral cover on reefs in the US Virgin Islands. Coral Reefs, 28: 925.

Mukherjee, B., P.N. Pandey and S.N. Singh (2002). Mathematical modelling and system analysis of inorganic carbon in the aquatic environment. Ecol. Modell., 152: 129-143.

Nürnberg, G. K. (2002). Quantification of oxygen depletion in lakes and reservoirs with the hypoxic factor. Lake Reserv. Manage., 18: 299-306.

O'Boyle, S., G. McDermott, J. Silke and C. Cusack (2016). Potential impact of an exceptional bloom of Karenia mikimotoi on dissolved oxygen levels in waters off western Ireland. Harmful Algae, 53: 77-85.

Parihar,L., V. Singh and J.K. Johal (2015). Bioremediation of uranium in contaminated water samples of Bathinda, Punjab by Clostridium sp. Res. J. Pharm. Bio. Chemi. Sci., 6: 509-513.

Patel, N., M. D. Khan, S. Shahane, D. Rai, D. Chauhan, C. Kant and V. K. Chaudhary (2020). Emerging pollutants in aquatic environment: source, effect, and challenges in Biomonitoring and Bioremediation-A Review. Pollution, 6: 99-113.

Rahul, K. Dhital, Risha, R. Bhattacharjee and G. Kumar (2018). Isolation of biosurfactant producing bacteria for oil cleaning from water bodies. Poll. Res., 37: 166-170.

Rashid, M., S. Kaur and R. Singh (2016). Fish health alterations as potential biomarkers of environmental quality in rivers of Punjab (India). Poll. Res., 35: 877-882.

Raven, J., K. Caldeira, H. Elderfield, O. Hoegh-Guldberg, P. Liss, U. Riebesell, J. Shepherd, C. Turley and A. Watson (2005). Ocean acidification due to increasing atmospheric carbon dioxide. The Royal Society.

Riebesell, U., N. Aberle-Malzahn, E.P. Achterberg, M. Algueró-Muñiz, S. Alvarez-Fernandez, J. Arístegui, L.T. Bach, M. Boersma, T. Boxhammer, W. Guan and M. Haunost (2018). Toxic algal bloom induced by ocean acidification disrupts the pelagic food web. Nat. Clim. Chang., 8:1082.

Sahoo, M. M. and K. C. Patra (2020). River water quality modelling and simulation based on Markov Chain Monte Carlo computation and Bayesian Inference model. African J. Sci. Technol. Innov. Dev.

Sarmento, H., J. M. Montoya, E. Vázquez-Domínguez, D. Vaqué and J. M. Gasol (2010). Warming effects on marine microbial food web processes: how far can we go when it comes to predictions? Philos. Trans. R. Soc. B Biol. Sci., 365: 2137-2149.

Sekerci, Y. and S. Petrovskii (2018). Global warming can lead to depletion of oxygen by disrupting phytoplankton photosynthesis: A mathematical modelling approach. Geosciences, 8: 201.

Shaffer, G., S.M. Olsen and J.O.P. Pedersen (2009). Long-term ocean oxygen depletion in response to carbon dioxide emissions from fossil fuels. Nat. Geosci., 2: 105.

Sharma, M., J. Singh, C. Baskar and A. Kumar (2018). A comprehensive review on biochar formation and its utilization for wastewater treatment. Poll. Res., 37: 1-18.

Sharma, H.B. and P.R. Sinha (2016). Performance analysis of vertical flow constructed wetland to treat domestic wastewater using two different filter media and canna as a plant. Indian J. Sci. Technol. 9: 1-7.

Shukla, B. K., A. R. Bhowmik, R. B. Raj and P. K. Sharma (2019). Physico-chemical parameters and status of ground water pollution in Jalandhar - Phagwara region. J. Green Eng., 9: 212-223.

Shukla, J. B., A. K. Misra and P. Chandra (2008). Modeling and analysis of the algal bloom in a lake caused by discharge of nutrients. Appl. Math. Comput., 196: 782-790.

Shukla, J. B., M. Verma and A. K. Misra (2017). Effect of global warming on sea level rise: A modeling study. Ecol. Complex., 32: 99-110.

Singh, J., S. Singh, S. Datta, J. Dutta, D.S. Dhanjal, A. Saini and J. Singh (2015). Toxicological effects of lambda-cyhalothrin on liver, kidney and testis of Indian catfish Clarias Batrachus. Toxicol. Int., 22: 128-136.

Singh, S., P. Sorokhaibam, R. Kaur and R. Singh (2018). Assessment of physicochemical parameters to investigate pollution status of white bein: A tributary of Sutlej river. Indian J. Ecol., 45: 898-900.

Solomon, S., G. K. Plattner, R. Knutti and P. Friedlingstein (2009). Irreversible climate change due to carbon dioxide emissions. Proc. Natl. Acad. Sci., 106: 1704-1709.

Stets, E. G., D. Butman, C.P. McDonald, S.M. Stackpoole, M.D. DeGrandpre and R. Striegl (2017). Carbonate buffering and metabolic controls on carbon dioxide in rivers. Global Biogeochem. Cycles, 31: 663-677.

Sweetman, A. K., A.R. Thurber, C.R. Smith, L.A. Levin, C. Mora, C.L. Wei, A.J. Gooday, D.O. Jones, M. Rex, M. Yasuhara and J. Ingels (2017). Major impacts of climate change on deep-sea benthic ecosystems. Elem. Sci. Anth., 5:1-23.

Vise, E., S. Das, R.K. Agarwal and A. Garg (2018). Identification of mycobacterium fortuitum isolated from diverse water sources in Punjab area. Poll. Res., 37: 47-53.

Winter, J. G., A.M. DeSellas, R. Fletcher, L. Heintsch, A. Morley, L. Nakamoto and K. Utsumi (2011). Algal blooms in Ontario, Canada: increases in reports since 1994. Lake Reserve. Manage., 27: 107-114.

Wolff, N. H., P. J. Mumby, M. Devlin and K. R. Anthony (2018). Vulnerability of the Great Barrier Reef to climate change and local pressures. Glob. Chang. Biol., 24: 1978-1991.

Yadav, R. (2019). Development of mathematical model to control human population. Plant Arch., 7: 49-57.

Yadav, R. and Priyanka (2019). Mathematical model describing effect of Sitotroga Cerealella on paddy in the environmental ecosystem. Plant Arch., 19: 2689-2692.

Yvon-Durocher, G., J.I. Jones, M. Trimmer, G. Woodward and J.M. Montoya (2010). Warming alters the metabolic balance of ecosystems. Philos. Trans. R. Soc. B Biol. Sci., 365: 2117-2126.