



ADDITIVE ALLOMETRIC MODELS OF SINGLE-TREE BIOMASS OF TWO-NEEDED PINES AS A BASIS OF REGIONAL MENSURATION STANDARDS FOR EURASIA

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Abstract

When using the unique in terms of the volumes of database on the level of a tree of the subgenus *Pinus* spp., the trans-Eurasian additive allometric model of biomass of trees for Eurasian forests are developed for the first time, and thereby the combined problem of model additivity and generality is solved. The additive model of tree biomass of *Pinus* is harmonized in two ways: it eliminated the internal contradictions of the component and the total biomass equations, and in addition, it takes into account regional differences of trees of equal sizes on total, aboveground and underground biomass. The proposed model and corresponding tables for estimating tree biomass makes them possible to calculate two-needled pine biomass (t/ha) on Eurasian forests when using measuring taxation.

Key words : Subgenus *Pinus* L., biomass of forests, allometric models, sample plots, biological productivity.

Introduction

In the conditions of continuously increasing biosphere functions of forest cover on our planet in recent years in world literature, dedicated to the problem of carbon-depositing ability of forests, there are two trends. The first of them relates to improving the correctness of biomass allometric equations, using of which the biological production of forests is estimated, in particular by ensuring the additivity of component composition (Parresol, 2001; Carvalho, 2003; Usoltsev, 2017; Usoltsev *et al.*, 2017b), and the second one is related to the need to develop global databases of actual data upon the biological productivity of forests with the development on their basis of global and transcontinental patterns, in connection with which the scientific community states the “big data era” coming (<http://www.gfbinitiative.org/symposium2017>) (Poorter *et al.*, 2015; Crowther *et al.*, 2015; Liang *et al.*, 2016; Jucker *et al.*, 2017).

Allometric models of tree biomass are harmonized or by ensuring the additivity of component composition

(Dong *et al.*, 2015), either by their regionalization (localization) using dummy variables (Fu *et al.*, 2012; Usoltsev *et al.*, 2017a) or by coding (marking) several tree species in a single model by dummy variables (Zeng, 2017) that is typically fulfilled on local data sets of tree biomass.

In this article the mentioned two approaches are joint, and the first attempt to develop additive allometric models of tree biomass of two-needled pines as a basis of regional taxation standards for Eurasia using compiled databases of tree biomass for forests of Eurasia (Usoltsev, 2016).

Materials and Methods

Of the mentioned database the materials in a number of 1700 sample trees of four vicarage species of the subgenus *Pinus* L. (*P. sylvestris* L., *P. tabulaeformis* Carr., *P. densiflora* S.et Z., *P. taeda* L.) are taken, that are distributed in nine eco-regions and marked respectively by nine dummy variables from X_0 to X_8 (table 1). The distribution of sample plots, on which sample trees are taken in different ecoregions of Eurasia is shown in fig. 1.

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Fig. 1 : The distribution of sample plots, on which sample trees are taken in different ecoregions of Eurasia. Red circles corresponds to natural stands, yellow ones – to plantations.

Table 1 : The scheme of regional coding actual biomass of 1700 sample trees two needed pines by dummy variables.

Region*	Species of <i>Pinus</i> L.	Dummy variables								Range of DBH, cm	Range of tree height, m	Number of measurements
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈			
WMÅ	<i>P. sylvestris</i>	0	0	0	0	0	0	0	0	1.4÷28.0	2.3÷27.0	66
ERn	<i>P. sylvestris</i>	1	0	0	0	0	0	0	0	0.9÷48.0	2.2÷19.6	60
ERs	<i>P. sylvestris</i>	0	1	0	0	0	0	0	0	1.0÷48.0	1.8÷32.6	291
Ural	<i>P. sylvestris</i>	0	0	1	0	0	0	0	0	2.4÷54.0	3.0÷30.2	278
WSm	<i>P. sylvestris</i>	0	0	0	1	0	0	0	0	0.5÷50.4	1.5÷28.8	270
WSfs	<i>P. sylvestris</i>	0	0	0	0	1	0	0	0	0.9÷38.0	1.7÷23.8	327
MS	<i>P. sylvestris</i>	0	0	0	0	0	1	0	0	0.8÷48.4	1.6÷26.8	377
Ch	<i>P. tabuliformis</i>	0	0	0	0	0	0	1	0	2.5÷18.0	3.3÷19.0	13
Jap	<i>P. densiflora</i> <i>P. taeda</i>	0	0	0	0	0	0	0	1	2.2÷24.0	2.0÷17.1	18

* Region designations: WME – West and Middle Europe; ERn – European part of Russia, the northern territory; ERs – European part of Russia, the southern territory; Ural – the Middle Ural; WSm - Western Siberia, middle and southern taiga; WSfs – Western Siberia, forest-steppe; MS – Middle Siberia, southern taiga; Ch – Northeast China; Jap – Japanese Islands.

According to the structure of disaggregating three-step additive model system (Tang *et al.*, 2000; Dong *et al.*, 2015), total biomass, estimated by the total equation, exploded into components according to the scheme presented in fig. 2. The coefficients of the regression models for all three steps are evaluated simultaneously, which ensures additivity of the all components: total, intermediate and initial ones (Dong *et al.*, 2015).

Results and Discussion

Initial allometric models are calculated

$$\ln P_i = a_i + b_i(\ln D) + c_i(\ln H) + d_i(\ln D)(\ln H) + \sum g_{ij} X_j \quad (1)$$

where, P_i – biomass of i -th component, kg; D – diameter on breast height, cm; H – tree height, m; i –

index of biomass component: total (t), aboveground (a), roots (r), tree crown (c), stem above bark (s), foliage (f), branches (b), stem wood (w) and stem bark (bk); j - index (code) of dummy variable, from 0 to 8 (table 1). $\sum g_{ij} X_j$ – block of dummy variables for i -th biomass component of j -th ecoregion. Model (1) after antilogarithmic procedure has the form

$$P_i = e^{a_i} D^{b_i} H^{c_i} D^{d_i(\ln H)} e^{\sum g_{ij} X_j} \quad (2)$$

Rationale for the structure of the regression model (1) was made earlier (Usoltsev *et al.*, 2017a). Since calculation of regression coefficients in the model (1) is made in the transformed data, to eliminate biases caused by logarithmic modification of variables, in the equation

Table 2 : The characteristic of independent allometric equations for two-needled pine trees.

Biomass component	Independent variables and the model regression coefficients										adjR ² *		
	D	H	D	H	D	H	D	H	D	H			
P _t	0.4544	D ^{1.7500}	H ^{-0.6328}	D ^{0.3068(lnD)}	e ^{-0.1168X1}	e ^{-0.2116X2}	e ^{-0.2818 X3}	e ^{-0.3533 X4}	e ^{-0.8662 X5}	e ^{-0.1548 X6}	e ^{-0.1300 X7}	e ^{0.1263 X8}	0.971
	0.1900	D ^{1.6828}	H ^{0.0360}	D ^{0.1751(lnD)}	e ^{-0.0400X1}	e ^{-0.1362X2}	e ^{-0.1348X3}	e ^{-0.2435X4}	e ^{-0.2644X5}	e ^{-0.2081X6}	e ^{-0.0941X7}	e ^{0.1707X8}	0.986
P _a	0.0876	D ^{2.0371}	H ^{-0.9146}	D ^{0.2950(lnD)}	e ^{-0.3567X1}	e ^{-0.2684X2}	e ^{-0.2624X3}	e ^{-0.0173X4}	e ^{-1.2534X5}	e ^{-0.1804X6}	e ^{-0.1892X7}	e ^{0.3160X8}	0.934
	0.2022	D ^{2.7353}	H ^{-1.7414}	D ^{0.1714(lnD)}	e ^{0.0099X1}	e ^{-0.0828X2}	e ^{-0.0751X3}	e ^{-0.1098X4}	e ^{-0.2266X5}	e ^{-0.2390X6}	e ^{-0.2774X7}	e ^{-0.1532X8}	0.935
P _c	0.0738	D ^{1.3743}	H ^{0.6740}	D ^{0.1575(lnD)}	e ^{-0.1159X1}	e ^{-0.1772X2}	e ^{-0.1514X3}	e ^{-0.2589X4}	e ^{-0.2441X5}	e ^{-0.1604X6}	e ^{-0.1497X7}	e ^{0.1848X8}	0.988
	0.0905	D ^{2.8821}	H ^{-1.6370}	D ^{0.0401(lnD)}	e ^{-0.1333X1}	e ^{0.0613X2}	e ^{-0.0084X3}	e ^{0.0225X4}	e ^{-0.0218X5}	e ^{-0.1684X6}	e ^{0.3424X7}	e ^{0.0946X8}	0.889
P _r	0.0752	D ^{2.7186}	H ^{-1.5747}	D ^{0.1993(lnD)}	e ^{-0.1175X1}	e ^{-0.1961X2}	e ^{-0.1454X3}	e ^{-0.2168X4}	e ^{-0.3791X5}	e ^{-0.3323X6}	e ^{-0.2576X7}	e ^{-0.2393X8}	0.939
	0.0377	D ^{1.3952}	H ^{0.8717}	D ^{0.1518(lnD)}	e ^{-0.2369X1}	e ^{-0.2595X2}	e ^{-0.1443X3}	e ^{-0.1584X4}	e ^{-0.1333X5}	e ^{0.1684X6}	e ^{-0.0151X7}	e ^{-0.5966X8}	0.991
P _w	0.0459	D ^{1.3640}	H ^{0.2267}	D ^{0.1198(lnD)}	e ^{-0.3582X1}	e ^{-0.8761X2}	e ^{-0.6960X3}	e ^{-0.6774X4}	e ^{-0.5021X5}	e ^{-0.5493X6}	e ^{-0.4182X7}	e ^{-0.3391X8}	0.967

* adjR² – adjusted coefficient of determination.

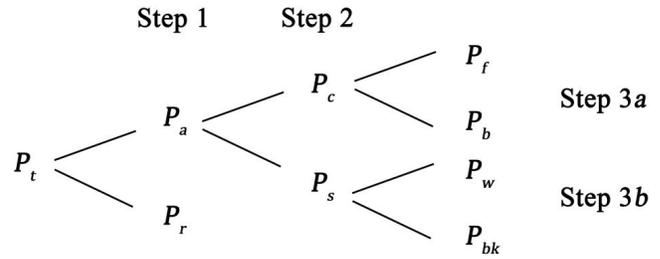


Fig. 2 : The pattern of disaggregating three-step proportional weighting additive model. Designations: P_t, P_r, P_a, P_c, P_s, P_f, P_b, P_w and P_{bk} are tree biomass respectively: total, underground (roots), aboveground, crown (needles and branches), stems above bark (wood and bark), needles, branches, stem wood and stem bark correspondingly, kg.

Table 3 : The structure of three-step additive models obtained by proportional weighting. Symbols here and further see in equation (1).

Step 1	$P = \frac{1}{1 + \dots} \times P$
Step 2	$P = \frac{1}{1 + \dots} \times P$
Step 3a	$P = \frac{1}{1 + \dots} \times P$
Step 3b	$P = \frac{1}{1 + \dots} \times P$

the amendment proposed by Baskerville (1972) is introduced. Using the programme of common regression analysis, the calculation of coefficients of equations (1) is performed and their characteristic is obtained, that is given in the table 2 after correcting the logarithmic transformation by G.L. Baskerville and bringing it to the form (2). All the regression coefficients for numerical variables of the equations (2) are significant at the level of probability of 0.95 or higher, and the equations are adequate to empirical data.

In accordance with the specifics of our study, the structure of the additive model, proposed by Chinese researchers (Tang et al., 2000; Dong et al., 2015) is modified. By substituting the regression coefficients of

Table 4 : Three-step additive model of component biomass composition for pine trees, obtained by proportional weighing.

$P_t = 0.4544 "D" ^{1.7500} "H" (^{-0.6328}) "D" ^{0.3068} (\ln H) ["e"] ^{-0.1168X1} "e" (^{-0.2116X2}) "e" (^{-0.2818X3}) "e" (^{-0.3533X4})$	
St	$P_a =$
ep	$"1" / ("1" + 0.4607 ["D"] ^{0.3543} ["H"] ^{0.9506} ["D"] ^{0.1199} (\ln H) ["e"] ^{-0.3967X1} ["e"] ^{-0.1322X2} ["e"]$
1	$\times P_t$
	$P_r =$
	$"1" / ("1" + 2.1706 ["D"] ^{-0.3543} ["H"] ^{0.9506} ["D"] ^{-0.1199} (\ln H) ["e"] ^{0.3967X1} ["e"] ^{0.1322X2} ["e"]$
	$\times P_t$
St	$P_c =$
ep	$"1" / ("1" + 0.3648 ["D"] ^{-1.3610} ["H"] ^{2.4154} ["D"] ^{-0.0139} (\ln H) ["e"] ^{0.1060X1} ["e"] ^{-0.0944X2} ["e"]$
2	$\times P_a$
	$P_s =$
	$"1" / ("1" + 2.7409 "D" ^{1.3610} "H" (^{-2.4154}) "D" ^{0.0139} (\ln H) ["e"] ^{-0.1060X1} ["e"] ^{0.0944X2} ["e"] ^{0.0763X3} ["e"]$
	$\times P_a$
St	$P_f =$
ep	$"1" / ("1" + 0.8312 ["D"] (^{-0.1636}) ["H"] ^{0.0623} "D" ^{0.1593} (\ln H) ["e"] ^{-0.2508X1} ["e"] (^{-0.2574X2}) ["e"] (^{-0.1370X3})$
3	$\times P_c$
a	$P_b =$
	$"1" / ("1" + 1.2031 "D" ^{0.1636} ["H"] (^{-0.0623}) ["D"] ^{-0.1593} (\ln H) ["e"] ^{0.2508X1} ["e"] ^{0.2574X2} ["e"] ^{0.1370X3}$
	$\times P_c$
St	$P_w =$
ep	$"1" / ("1" + 1.2186 ["D"] ^{-0.0311} ["H"] ^{-0.6450} ["D"] (^{-0.0320} (\ln H)) ["e"] (^{-0.5952X1}) ["e"] (^{-0.6166X2}) ["e"] (^{-0.5517X3})$
3	$\times P_s$
b	$P_b k =$
	$"1" / ("1" + 0.8206 ["D"] ^{0.0311} ["H"] ^{0.6450} ["D"] ^{0.0320} (\ln H) ["e"] ^{0.5952X1} ["e"] ^{0.6166X2} ["e"] ^{0.5517X3} ["e"]$
	$\times P_s$

initial equations from table 2 into the structure of the additive model, presented in table 3, when using three-step scheme of proportional weighting, we got transcontinental additive model of component composition of pine tree biomass of double harmonization, the final appearance of which is given in table 4. The model is valid in the range of actual data of height and diameter of the sample trees shown in table 1.

By tabulating the model obtained (table 4) according to the given values of *D* and *H* as well as by the values of the dummy variables, localizing the general model for eco-regions, you can calculate regional transcontinental standards for Eurasia, intended for estimating tree and forest additive biomass components.

Because sometimes it is impossible to measure the height of trees in sample plots, for such cases when calculating the biomass per ha the auxiliary equation (3) intended for using the proposed model (2) is calculated;

$$H = 2,196 D^{0.7011} e^{-0.0094X1} e^{0.0725X2} e^{0.1138X3} e^{0.0145X4} e^{-0.1679X5} e^{-0.0164X6} e^{-0.1681X7} e^{-0.3390X8}; \tag{3}$$

$$adjR^2 = 0,888.$$

Tabulating of built additive models (2) in Excel format is fulfilled. Because the volume of tables obtained can exceed the format of journal article, we limit ourselves to some regional characteristics analysis of the structure of biomass of trees of the same size when using the fragment of summary table for two-needled pines (table 5). In their analysis, you can see that the biomass of all biomass

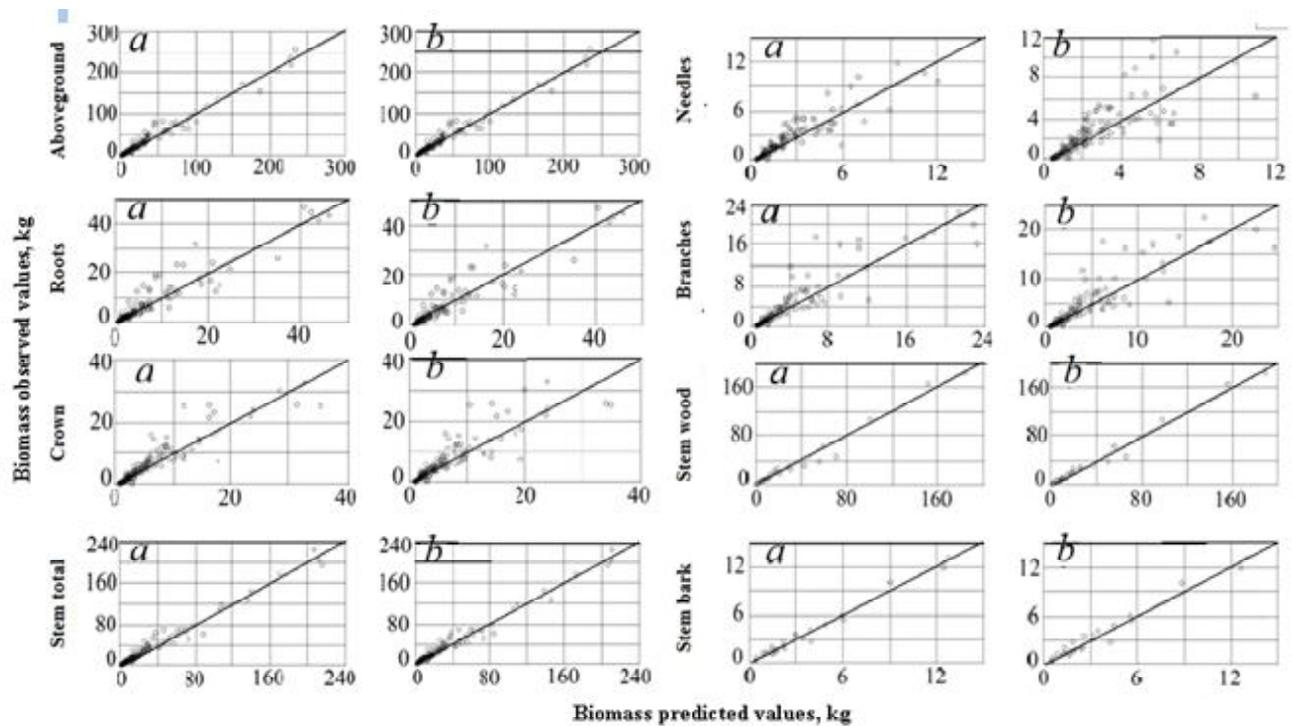


Fig. 3 : The ratio of observed values and the values derived by calculation of independent (a) and additive (b) models of tree biomass.

Table 5 : Fragments of the table of additive tree biomass for diameter 14 cm and tree height of 14 m according to the eco-regions and corresponding species of the subgenus *Pinus* L.

Biomass component	Eco-regions and corresponding species of the subgenus <i>Pinus</i>								
	WME <i>P. sylvestris</i>	ERn <i>P. sylvestris</i>	ERs <i>P. sylvestris</i>	Ural <i>P. sylvestris</i>	WSm <i>P. sylvestris</i>	WSfs <i>P. sylvestris</i>	MS <i>P. sylvestris</i>	Ch <i>P. tabulaeformis</i>	Jap <i>P. densiflora</i> <i>P. taeda</i>
Total biomass	73.44	65.35	59.44	55.41	51.59	30.89	62.91	64.49	83.33
Roots	13.25	8.42	9.61	8.99	11.46	2.34	11.61	10.75	16.90
Aboveground	60.20	56.92	49.83	46.42	40.13	28.55	51.30	53.74	66.43
Tree crown	9.48	8.19	8.49	7.79	7.15	4.56	7.55	11.96	10.18
Foliage	3.23	3.27	3.41	2.90	2.84	1.94	2.86	4.31	3.15
Branches	6.24	4.91	5.08	4.88	4.31	2.62	4.69	7.65	7.03
Stem above bark	50.72	48.73	41.34	38.63	32.98	23.99	43.75	41.78	56.25
Stem wood	43.59	44.70	37.98	35.30	30.05	21.55	40.51	37.66	52.85
Stem bark	7.14	4.04	3.36	3.33	2.93	2.44	3.24	4.12	3.40

components of pine trees of equal size is dropping in the direction from the Pacific and Atlantic coasts to the Siberian regions.

It was found (Cunia and Briggs, 1984; Reed and Green, 1985), that the correction of internal inconsistency of biomass equations by ensuring their additivity does not necessarily mean improvements in the accuracy of biomass estimating. Therefore, it is necessary to ascertain, whether adequate the additive model obtained and how its adequacy characteristics are related to the same indices of independent (trivial) equations?

To this purpose, the estimates of biomass obtained from independent and additive equations are compared with actual biomass values by calculating the coefficient of determination R^2 calculated by the formula;

$$R = 1 - \frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \tag{4}$$

where Y_i - actual biomass values; \hat{Y}_i - predicted biomass values; \bar{Y} - the mean actual value of all (N) trees.

Table 6 : The characteristics of “methodized” independent allometric equations for two-needled pine trees.

Biomass component	Components of regression models											
	P_t	P_a	P_r	P_s	P_w	P_{bk}	P_c	P_b	P_f			
P_t	0.4544	$D^{1.7500}$	$H^{-0.6328}$	$D^{0.3068}(ln/t)$	$e^{-0.1168X1}$	$e^{-0.2116 X2}$	$e^{-0.2818 X3}$	$e^{-0.3533 X4}$	$e^{-0.8662 X5}$	$e^{-0.1548 X6}$	$e^{-0.1300 X7}$	$e^{0.1263 X8}$
P_a	0.3484	$D^{1.6835}$	$H^{-0.5342}$	$D^{0.3033}(ln/t)$	$e^{-0.0557 X1}$	$e^{-0.1943 X2}$	$e^{-0.2693 X3}$	$e^{-0.4355 X4}$	$e^{-0.7667 X5}$	$e^{-0.1403 X6}$	$e^{-0.0991 X7}$	$e^{0.0847 X8}$
0.0876	$D^{2.0371}$	$H^{-0.9146}$	$D^{0.2950}(ln/t)$	$e^{-0.5567 X1}$	$e^{-0.2684 X2}$	$e^{-0.2624 X3}$	$e^{0.0173 X4}$	$e^{-1.2534 X5}$	$e^{-0.1804 X6}$	$e^{-0.1892 X7}$	$e^{0.3160 X8}$	
P_c	0.4699	$D^{2.5219}$	$H^{-2.3704}$	$D^{0.3768}(ln/t)$	$e^{-0.2397 X1}$	$e^{0.0518 X2}$	$e^{-0.3689 X3}$	$e^{-0.6157 X4}$	$e^{-0.9318 X5}$	$e^{-0.1914 X6}$	$e^{0.2307 X7}$	$e^{0.0216 X8}$
P_s	0.1036	$D^{1.3760}$	$H^{0.3002}$	$D^{0.2493}(ln/t)$	$e^{0.0398 X1}$	$e^{-0.2229 X2}$	$e^{-0.2209 X3}$	$e^{-0.3153 X4}$	$e^{-0.5733 X5}$	$e^{-0.1132 X6}$	$e^{-0.1298 X7}$	$e^{0.1611 X8}$
P_f	0.3392	$D^{2.6434}$	$H^{-2.7239}$	$D^{0.3623}(ln/t)$	$e^{-0.4110 X1}$	$e^{0.2064 X2}$	$e^{-0.3144 X3}$	$e^{-0.6026 X4}$	$e^{-0.9611 X5}$	$e^{0.0900 X6}$	$e^{0.2690 X7}$	$e^{-0.1211 X8}$
P_b	0.1494	$D^{2.4959}$	$H^{-1.9959}$	$D^{0.3472}(ln/t)$	$e^{-0.1448 X1}$	$e^{-0.0568 X2}$	$e^{-0.4330 X3}$	$e^{-0.6008 X4}$	$e^{-0.8704 X5}$	$e^{-0.4264 X6}$	$e^{0.2111 X7}$	$e^{0.1217 X8}$
P_w	0.0377	$D^{1.3952}$	$H^{0.8717}$	$D^{0.1518}(ln/t)$	$e^{0.2369 X1}$	$e^{-0.2595 X2}$	$e^{-0.1443 X3}$	$e^{-0.1584 X4}$	$e^{-0.1333 X5}$	$e^{0.1684 X6}$	$e^{-0.0151 X7}$	$e^{0.5966 X8}$
P_{bk}	0.0459	$D^{1.3640}$	$H^{0.2267}$	$D^{0.1198}(ln/t)$	$e^{-0.3582 X1}$	$e^{-0.8761 X2}$	$e^{-0.6960 X3}$	$e^{-0.6774 X4}$	$e^{-0.5021 X5}$	$e^{-0.5493 X6}$	$e^{-0.4182 X7}$	$e^{-0.3391 X8}$

Table 7 : Comparison of the adequacy indices of the independent and additive equations for larch tree biomass pines.

Adequacy index	Biomass components *								
	P_t	P_a	P_r	P_s	P_w	P_{bk}	P_c	P_b	P_f
	Independent (initial) equations								
R^2	0.945	0.947	0.765	0.953	0.886	0.972	0.760	0.766	0.716
	Additive equations								
R^2	0.945	0.950	0.755	0.958	0.955	0.962	0.718	0.734	0.557

*Designations see fig. 2 and equation (1). Bold components, for which R^2 values of the additive models higher than independent ones.

To properly compare the adequacy of independent and additive equations, we modify the original data to a comparable condition, *i.e.* independent equations for all components of biomass are calculated according to the same data that the additive ones and the equations for the total biomass. Description of such “methodized” equations is given in table 6. The results of the comparison (table 7) indicate that while additive equations internally consistent, but compared to the independent equations they have better characteristics of adequacy not for all component biomass. This corresponds to the view (Cunia and Briggs, 1984; Reed and Green, 1985), that the correction of internal inconsistency of biomass equations by ensuring their additivity does not necessarily means improvements in the accuracy of biomass estimating.

The ratio of actual values and derived ones by tabulating independent and additive tree biomass models (fig. 3) shows the degree of correlativeness of the actual and calculated values and, in many cases, the absence of visible differences in the structure of residual variances obtained on two named models. More or less the value of R^2 of one or the other model is determined by the random position of actual values of biomass of largest trees in confidence range and uneven dispersion, namely accidental because of their small number and the greatest contribution to the residual variance (fig. 3).

Conclusion

When using the unique in terms of the volumes of database on the level of a tree of the subgenus *Pinus* spp., the trans-Eurasian additive allometric model of biomass of trees for Eurasian forests are developed for the first time, and thereby the combined problem of model additivity and generality is solved. The additive model of tree biomass of *Pinus* is harmonized in two ways: it eliminated the internal contradictions of the component and the total biomass equations, and in addition, it takes into account regional differences of trees of equal sizes on total, aboveground and underground biomass. The proposed model and corresponding tables for estimating tree biomass makes them possible to

calculate two-needled pine biomass (t/ha) on Eurasian forests when using measuring taxation.

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